Exercice 1

a) $\sqrt{x^4-6x^2} = 3\sqrt{x}$ | $()^2$

$x^4-6x^2 = 9x$ | $-9x$

$x^4-6x^2-9x = 0$

$x(\underbrace{x^3-6x-9}_{P}) = 0$

$p(3) = 27-18-9 = 0 \Rightarrow (x-3) / P$

Par Horner :

	1	0	-6	-9
3		3	9	9
	1	3	3	0

L'équation :

$x(x-3)(\underbrace{x^2+3x+3}_{\Delta = 9-12<0}) = 0$

$\Rightarrow x = 0$ ou $x = 3$

Preuve : $x=0$: $0 = 0$ ✓ ok

$x=3$: $\sqrt{81-54} \overset{?}{=} 3\sqrt{3}$

$\sqrt{27} = \sqrt{27}$ ✓ ok

$S_a = \{0 ; 3\}$

b) $\sqrt{4x-13} = -2x+1 \quad | \; ()^2$

$4x-13 = 4x^2-4x+1$

$4x^2-8x+14=0$

$2x^2-4x+7=0$

$\Delta = 16-56 < 0$

$S_b = \emptyset$

c) $\sqrt{7x-3} - \sqrt{x+5} = \sqrt{3x-8} \quad | \; ()^2$

$7x-3-2\sqrt{(7x-3)(x+5)}+x+5 = 3x-8$

$5x+10 = 2\sqrt{} \quad | \; ()^2$

$25x^2+100x+100 = 4(7x^2+32x-15)$

$25x^2+100x+100 = 28x^2+128x-60$

$3x^2+28x-160=0$

$\Delta = 28^2+1920 = 2704 = 52^2$

$x_1 = \frac{-28-52}{6} = \frac{-80}{6} = -\frac{40}{3}$

$x_2 = \frac{-28+52}{6} = \frac{24}{6} = 4$

<u>Preuve</u> : $x_1 = -40/3$, $\sqrt{\underbrace{7x-3}_{<0}}$ - .... impossible

$x_2 = 4$: $\underbrace{\sqrt{28-3}}_{5} - \sqrt{4+5} \overset{?}{=} \underbrace{\sqrt{12-8}}_{2}$

$5 - 3 = 2$ ✓ ok

$S_t = \{4\}$

d) $\sqrt{x+3} + \sqrt{x+8} = 5\sqrt{x}$ $\quad | ()^2$

$x+3 + 2\sqrt{\ }\sqrt{\ } + x + 8 = 25x$

$2\sqrt{\ }\sqrt{\ } = 23x - 11$ $\quad | ()^2$

$4(x^2 + 11x + 24) = 529x^2 - 506x + 121$

$525x^2 - 550x + 25 = 0$ $\quad | \div 25$

$21x^2 - 22x + 1 = 0$

$(21x - 1)(x - 1) = 0$

$x = \frac{1}{21}$ <u>ou</u> $x = 1$

<u>Preuve</u> : 1) $x = 1/21$ gauche : $\sqrt{\frac{1}{21}+3} + \sqrt{\frac{1}{21}+8} = \sqrt{\frac{64}{3}} + \sqrt{\frac{169}{3}}$

$= \frac{8}{\sqrt{3}} + \frac{13}{\sqrt{3}} = \frac{21}{\sqrt{3}}$

droite : $5 \cdot \frac{1}{\sqrt{21}} = \frac{5}{\sqrt{21}}$, ne convient pas

2) $x = 1$: $2 + 3 = 5$ ✓ convient

$S_d = \{1\}$

e) $2x - 20 = -2\sqrt{2x^2 + x}$ $\quad \Big| \div 2$

$x - 10 = -\sqrt{2x^2 + x}$ $\quad \Big| \,()^2$

$x^2 - 20x + 100 = 2x^2 + x$

$x^2 + 21x - 100 = 0$

$(x + 25)(x - 4) = 0$

$x = -25$ ou $x = 4$

<u>Vérification:</u> 1) $x = -25$:

gauche: $-50 + 1 + 2\sqrt{-25(-50+1)} = -49 + 2\sqrt{-25 \cdot (-49)}$

$= -49 + 70 = +21$ ok

2) $x = 4$:

gauche: $8 + 1 + 2\sqrt{4 \cdot 9} = 9 + 12 = 21$ ok

$S_e = \{-25; 4\}$

f) $\sqrt{7x-27} = \sqrt{2x+1} + \sqrt{3x+4} \quad \Big| \, ()^2$

$7x-27 = 5x+5+2\sqrt{\ }\sqrt{\ }$

$2x-32 = 2\sqrt{\ }\sqrt{\ } \quad \Big| \div 2$

$x-16 = \sqrt{\ }\sqrt{\ } \quad \Big| \, ()^2$

$x^2-32x+256 = 6x^2+11x+4$

$5x^2+43x-252 = 0$

$\Delta = 6889 = 83^2$

$x_1 = \frac{-43-83}{10} = \frac{-126}{10} = \frac{-63}{5}$; $x_2 = \frac{-43+83}{10} = 4$

Vérif: 1) $x_1 = \frac{-63}{5}$ gauche : $\sqrt{\underbrace{\frac{-441}{5}-27}_{<0}}$ impossible

2) $x_2 = 4$: gauche : $\sqrt{28-27} = 1$

droite: $\sqrt{8+1} + \sqrt{12+4} = 3+4 = 7$ } ne convient pas!

$S_f = \emptyset$

$\left[\underbrace{x-16}_{-12} = 2\sqrt{2x+1}\cdot\sqrt{3x+4} \right]$

$-12 = 12$!