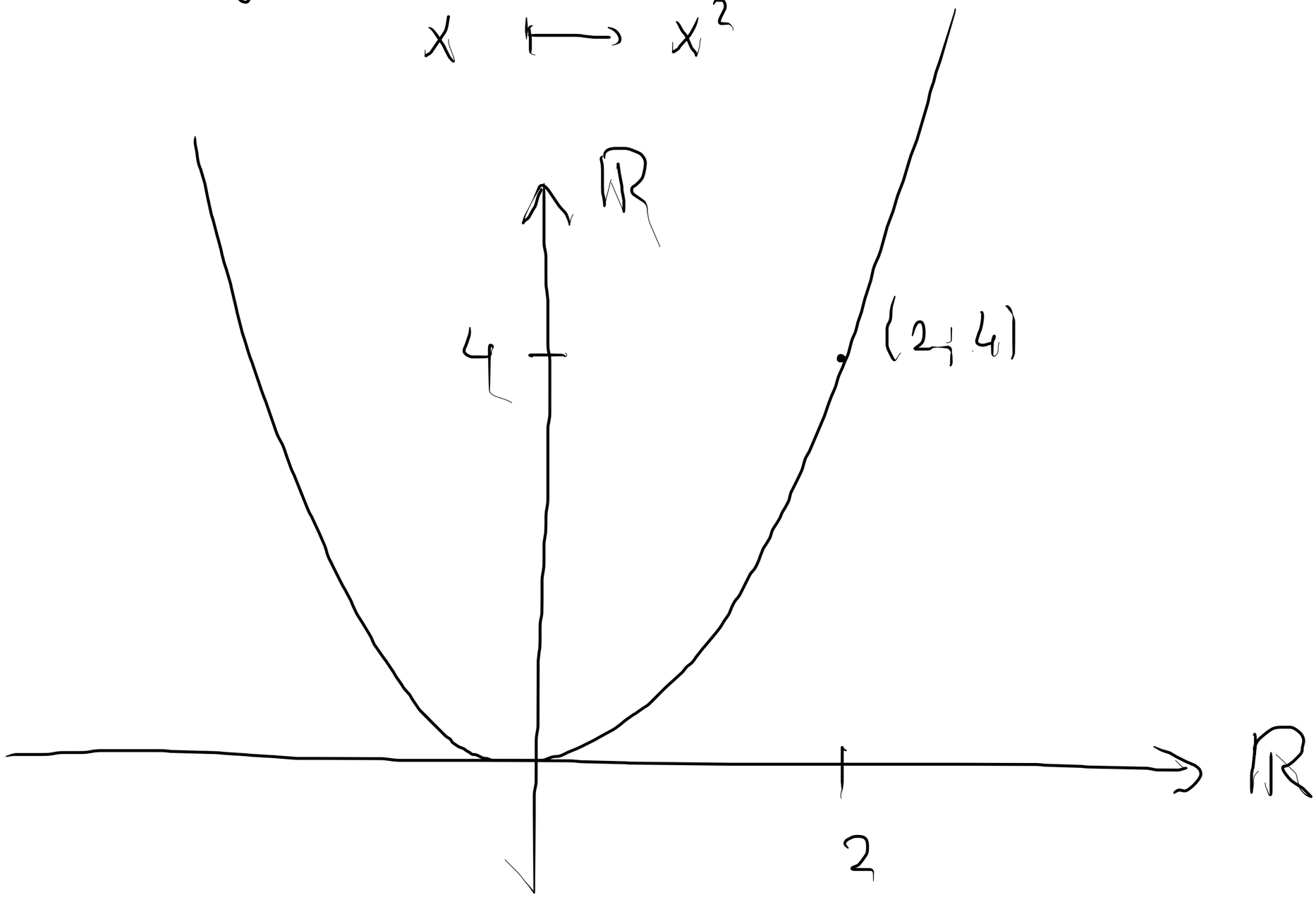
$$f : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto x^2$$



2.5.19 Résoudre les systèmes d'équations :

a) $\begin{cases} x+y &= 1 \\ x-y &= 0 \end{cases}$

b) $\begin{cases} 5x-2y &= 5 \\ 3x-y &= 10 \end{cases}$

c) $\begin{cases} 6x+4 &= -6y \\ 1-x &= 6y \end{cases}$

a) ① $x + y = 1$ est une équation

$$S_1 = \left\{ (1;0) ; (0,1) ; \left(\frac{1}{2}, \frac{1}{2}\right) ; (0.4 ; 0.6) ; \left(\frac{1}{3} ; \frac{2}{3}\right) ; (4 ; -3) ; \dots \right\}$$

$$= \{ (x, y) \mid x + y = 1 \}$$

② $x - y = 0$

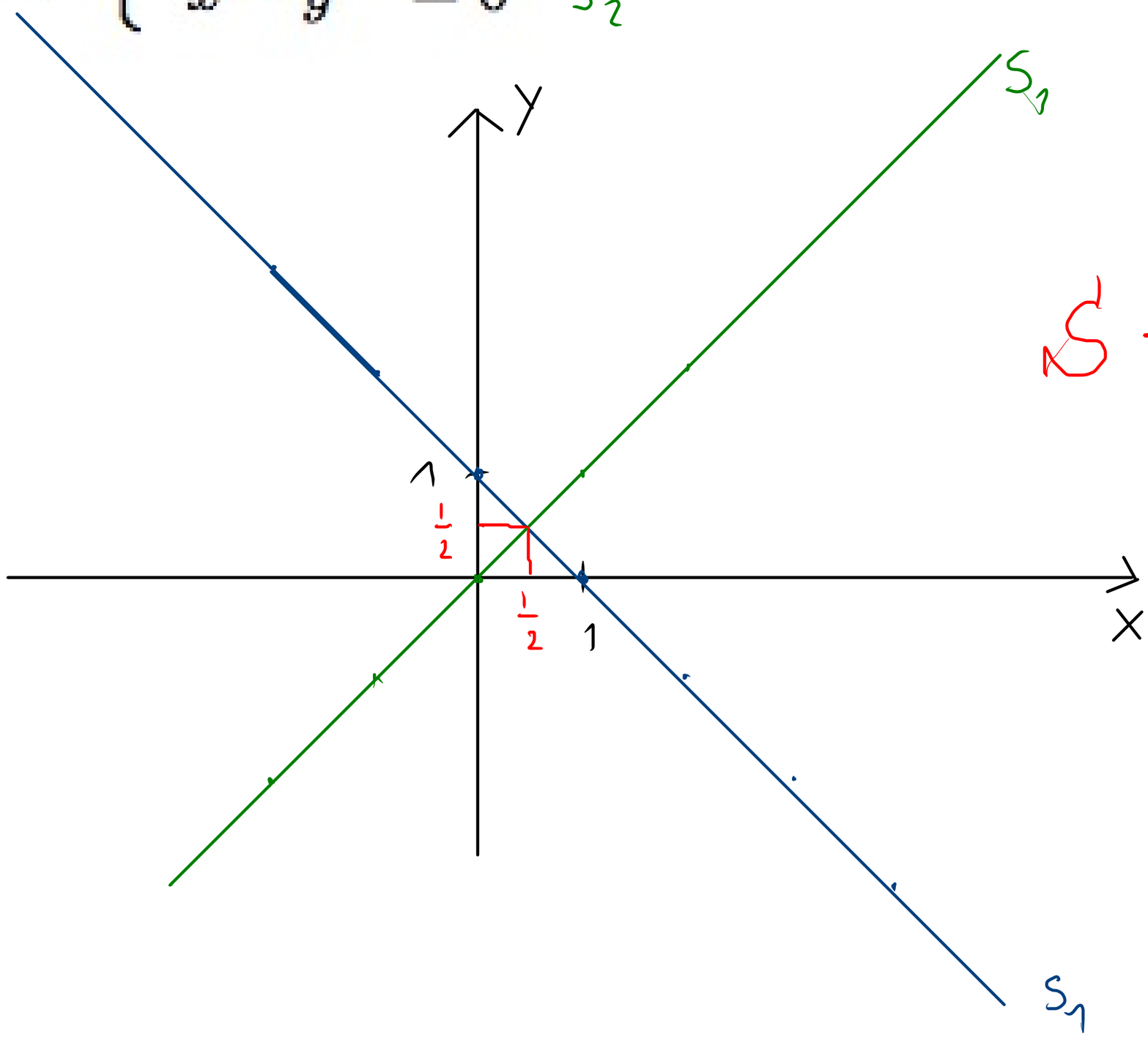
$$S_2 = \left\{ (1;1) ; (2;2) ; \left(\frac{1}{2} ; \frac{1}{2}\right) ; (-3 ; -3) ; \dots \right\}$$

$$= \{ (x, y) \mid x - y = 0 \}$$

Donc, il semble que $S = \left\{ \left(\frac{1}{2} ; \frac{1}{2}\right) \right\}$

Par voie graphique :

a) $\begin{cases} x + y = 1 & S_1 \\ x - y = 0 & S_2 \end{cases}$



$S = \left\{ \left(\frac{1}{2} ; \frac{1}{2} \right) \right\}$

$$3(x+1) = 3x+3$$

$$3x+3 = 3x+3$$

$$0 = 0$$

$$S = \mathbb{R}$$

a) $\begin{cases} x + y &= 1 \\ x - y &= 0 \end{cases} \left| \begin{array}{l} L_1 \\ L_2 \leftarrow L_2 - L_1 \end{array} \right.$

$\Leftrightarrow \begin{cases} x + y = 1 \\ \quad -2y = -1 \end{cases} \left| \begin{array}{l} L_1 \\ L_2 \leftarrow -\frac{1}{2} L_2 \end{array} \right.$

$\Leftrightarrow \begin{cases} x + y = 1 \\ \quad y = \frac{1}{2} \end{cases} \left| \begin{array}{l} L_1 \leftarrow L_1 - L_2 \\ L_2 \end{array} \right.$

$\Leftrightarrow \begin{cases} x = \frac{1}{2} \\ y = \frac{1}{2} \end{cases}$

b) $\begin{cases} 5x - 2y &= 5 \\ 3x - y &= 10 \end{cases}$ $\quad \Big| \quad \begin{array}{l} L_1 \leftarrow 3L_1 \\ L_2 \leftarrow 5L_2 \end{array}$

$$\Leftrightarrow \begin{cases} 15x - 6y = 15 \\ 15x - 5y = 50 \end{cases} \quad \Big| \quad \begin{array}{l} L_1 \leftarrow \frac{1}{3} L_1 \\ L_2 \leftarrow L_2 - L_1 \end{array}$$

$$\Leftrightarrow \begin{cases} 5x - 2y = 5 \\ y = 35 \end{cases} \quad \Big| \quad L_1 \leftarrow L_1 + 2L_2$$

$$\Leftrightarrow \begin{cases} 5x = 75 \\ y = 35 \end{cases} \quad \Big| \quad L_1 \leftarrow \frac{1}{5} L_1$$

$$\Leftrightarrow \begin{cases} x = 15 \\ y = 35 \end{cases}$$

c) $\begin{cases} 6x+4 &= -6y \\ 1-x &= 6y \end{cases}$

$$\Leftrightarrow \begin{cases} 6x + 6y = -4 \\ -x - 6y = -1 \end{cases} \quad \Big| \quad L_1 \leftrightarrow L_2$$

$$\Leftrightarrow \begin{cases} -x - 6y = -1 \\ 6x + 6y = -4 \end{cases} \quad \Big| \quad \begin{array}{l} L_1 \leftarrow -L_1 \\ L_2 \leftarrow \frac{1}{2} L_2 \end{array}$$

$$\Leftrightarrow \begin{cases} \boxed{x} + 6y = 1 \\ \boxed{3x} + 3y = -2 \end{cases} \quad \Big| \quad L_2 \leftarrow L_2 - 3L_1$$

$$\Leftrightarrow \begin{cases} x + 6y = 1 \\ -15y = -5 \end{cases} \quad \Big| \quad L_2 \leftarrow -\frac{1}{15} L_2$$

$$\Leftrightarrow \begin{cases} x + 6y = 1 \\ y = \frac{1}{3} \end{cases} \quad \Big| \quad L_1 \leftarrow L_1 - 6L_2$$

$$\Leftrightarrow \begin{cases} x = -1 \\ y = \frac{1}{3} \end{cases}$$

d) $\begin{cases} 2x - 4y &= 2 \\ x - 2y &= 1 \end{cases}$ $\Big|$ $L_1 \leftarrow \frac{1}{2} L_1$

deux droites identiques

$$\Leftrightarrow \begin{cases} x - 2y = 1 \\ x - 2y = 1 \end{cases}$$

$$\Leftrightarrow \quad x - 2y = 1$$

$y = \frac{x-1}{2}$

$x = 2y + 1$

$$S = \{ (x, y) \mid x - 2y = 1 \}$$

$$= \{ (2y + 1, y) \}$$

$$= \{ (x, \frac{x-1}{2}) \}$$

e) $\begin{cases} 2x + 4y &= 5 \\ x + 2y &= 2 \end{cases}$ $\quad L_1 \leftrightarrow L_2$

deux droites parallèles

$$\Leftrightarrow \begin{cases} x + 2y = 2 \\ 2x + 4y = 5 \end{cases} \quad L_2 \leftarrow L_2 - 2L_1$$

$$\Leftrightarrow \begin{cases} x + 2y = 2 \\ 0 = 1 \end{cases}$$

$S = \emptyset$

Méthode par élimination

$$\begin{cases} x - 2y + 3z = 4 \\ \boxed{2x} + y - 4z = 3 \\ \boxed{-3x} + 4y - z = -2 \end{cases} \quad \left| \begin{array}{l} \\ L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 + 3L_1 \end{array} \right.$$

$$\Leftrightarrow \begin{cases} x - 2y + 3z = 4 \\ 5y - 10z = -5 \\ -2y + 8z = 10 \end{cases} \quad \left| \begin{array}{l} \\ L_2 \leftarrow \frac{1}{5} L_2 \\ L_3 \leftarrow -\frac{1}{2} L_3 \end{array} \right.$$

$$\begin{cases} x - 2y + 3z = 4 \\ y - 2z = -1 \\ \boxed{y} - 4z = -5 \end{cases} \quad \left| \begin{array}{l} \\ \\ L_3 \leftarrow L_3 - L_2 \end{array} \right.$$

$$\Leftrightarrow \begin{cases} x - 2y + 3z = 4 \\ y - 2z = -1 \\ -2z = -4 \end{cases} \quad \left| \begin{array}{l} \text{Système échelonné} \\ \\ L_3 \leftarrow -\frac{1}{2} L_3 \end{array} \right.$$

$$\Leftrightarrow \begin{cases} \boxed{x} - 2y + 3z = 4 \\ \boxed{y} - 2z = -1 \\ \boxed{z} = 2 \end{cases} \quad \left| \begin{array}{l} L_1 \leftarrow L_1 - 3L_3 \\ L_2 \leftarrow L_2 + 2L_3 \\ \\ \end{array} \right.$$

$$\Leftrightarrow \begin{cases} x - 2y = -2 \\ y = 3 \\ z = 2 \end{cases} \quad \left| \begin{array}{l} L_1 \leftarrow L_1 + 2L_2 \\ \\ \\ \end{array} \right.$$

$$\Leftrightarrow \begin{cases} x = 4 \\ y = 3 \\ z = 2 \end{cases}$$

Système échelonné réduit

g) $\begin{cases} 2x + 3y + 2z = 41 \\ 8x + 5y \qquad = 31 \\ \qquad 7y \qquad = 21 \end{cases}$

$$\Leftrightarrow \begin{cases} 8x + 5y \qquad = 31 \\ \qquad 7y \qquad = 21 \\ 2x + 3y + 2z = 41 \end{cases} \quad \left| \begin{array}{l} L_2 \leftarrow \frac{1}{7} L_2 \\ L_3 \leftarrow 4L_3 - L_1 \end{array} \right.$$

$$\Leftrightarrow \begin{cases} 8x + 5y = 31 \\ \qquad y = 3 \\ \qquad 7y + 8z = 133 \end{cases} \quad \left| \begin{array}{l} L_1 \leftarrow L_1 - 5L_2 \\ \\ L_3 \leftarrow L_3 - 7L_2 \end{array} \right.$$

$$\Leftrightarrow \begin{cases} 8x = 16 \\ \qquad y = 3 \\ \qquad 8z = 112 \end{cases} \quad \left| \begin{array}{l} L_1 \leftarrow \frac{1}{8} L_1 \\ \\ L_3 \leftarrow \frac{1}{8} L_3 \end{array} \right.$$

$$\begin{cases} x = 2 \\ \qquad y = 3 \\ \qquad z = 14 \end{cases}$$