

Algèbre I – Auto évaluation 2**Exercice 1**

Résoudre les deux équations suivantes

a) $x = \sqrt{7x+29} - 3$

b) $8 = 2x - \sqrt{x^2 - 13}$

a) $x + 3 = \sqrt{7x+29}$

$x^2 + 6x + 9 = 7x + 29$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

Vérification :

(i) $x = 5$:

$5 + 3 = \sqrt{7 \cdot 5 + 29}$
8 = $\sqrt{64}$

8 = 8 on ✓

(ii) $x = -4$

$-4 + 3 = \sqrt{-28 + 29}$
-1 = 1

ne convient pas !

$S = \{5\}$

$(\)^2$

b) $2x - 8 = \sqrt{x^2 - 13}$

Δ

$4x^2 - 32x + 64 = x^2 - 13$

$3x^2 - 32x + 77 = 0$

$\Delta = (-32)^2 - 4 \cdot 3 \cdot 77 =$

$1024 - 924 = 100 = 10^2$

$x_1 = \frac{32 - 10}{6} = \frac{22}{6} = \frac{11}{3}$

$x_2 = \frac{32 + 10}{6} = \frac{42}{6} = 7$

Vérification :

(i) $x = \frac{11}{3}$:

$2 \cdot \frac{11}{3} - 8 = \sqrt{\left(\frac{11}{3}\right)^2 - 13}$

$\frac{22}{3} - 8 = \sqrt{\frac{121}{9} - 13}$

$-\frac{2}{3} = \sqrt{\frac{121 - 117}{9}}$

$-\frac{2}{3} = \frac{2}{3}$

ne convient pas !

(ii) $x = 7$:

$6 = \sqrt{49 - 13}$

$6 = 6$ on ✓

Exercice 2

Résoudre les deux équations suivantes

$$a) \frac{x-4}{x} + \frac{1}{x-4} = \frac{4}{x^2 - 4x}$$

$$b) \frac{2x+1}{x^2 + 14x + 48} - \frac{x+2}{x^2 + 15x + 56} + \frac{5}{x^2 + 13x + 42} = 0$$

$$a) \frac{x-4}{x} + \frac{1}{x-4} = \frac{4}{x(x-4)}$$

$$ED = \mathbb{R} - \{0; 4\}$$

$$(x-4)^2 + x = 4$$

$$x^2 - 8x + 16 + x - 4 = 0$$

$$x^2 - 7x + 12 = 0$$

$$(x-4)(x-3) = 0 \Rightarrow x=4 \notin ED \text{ ou } x=3; S = \{3\}$$

$$b) \frac{2x+1}{(x+6)(x+8)} - \frac{x+2}{(x+7)(x+8)} + \frac{5}{(x+6)(x+7)} = 0 \quad ED = \mathbb{R} - \{-6; -7; -8\}$$

$$(2x+1)(x+7) - (x+2)(x+6) + 5(x+8) = 0$$

$$2x^2 + 15x + 7 - x^2 - 8x - 12 + 5x + 40 = 0$$

$$x^2 + 12x + 35 = 0$$

$$(x+7)(x+5) = 0$$

$$x=-7 \notin ED \text{ (ne convient pas)}, \quad x=-5 \in ED$$

$$S = \{-5\}$$

Exercice 3

Résoudre les deux systèmes d'équations.

$$a) \begin{cases} x + 2y + 3z = 1 \\ 4x + 5y + 5z = 2x \\ 7x + 8y + 9z = 3y + 4 \end{cases}$$

$$\left| \begin{array}{l|l|l} & x & x \\ \begin{cases} x + 2y + 3z = 1 \\ 2x + 5y + 5z = 0 \\ 7x + 8y + 9z = 4 \end{cases} & \cdot 2 & \cdot (-1) \\ & & & \cdot (-1) \end{array} \right|$$

$$b) \begin{cases} x^2 + y^2 - 2x - 4y = 20 \\ 3x - y = 6 \end{cases}$$

Par substitution:

$$\begin{cases} y = 3x - 6 \end{cases}$$

$$(*) \begin{cases} x^2 + (3x-6)^2 - 2x - 4(3x-6) = 20 \end{cases}$$

Réolution (*) :

$$\begin{cases} x^2 + 9x^2 - 36x + 36 - 2x - 12x + 24 = 20 \end{cases}$$

$$10x^2 - 50x + 40 = 0 \quad | :10$$

$$x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) = 0$$

deux solutions :

$$\begin{cases} x=4, y=6 \end{cases}$$

$$\begin{cases} x=1, y=-3 \end{cases}$$

$$S = \{(4;6), (1;-3)\}$$

$$\Leftrightarrow \left| \begin{array}{l|l|l} & y & z \\ \begin{cases} -y + \frac{z}{2} = 2 \\ 9y + 12z = 3 \\ x + 2y + 3z = 1 \end{cases} & \cdot 9 & \cdot 12 \\ & 1 & \cdot (-1) \end{array} \right|$$

$$\Leftrightarrow \left| \begin{array}{l} 21z = 21 \\ -21y = 21 \\ x + 2y + 3z = 1 \end{array} \right|$$

$$\left| \begin{array}{l} z = 1 \\ y = -1 \\ x - 2 + 3 = 1 \end{array} \right|$$

$$\Leftrightarrow x = 0, y = -1 \text{ ou } z = 1$$

$$S = \{(0,-1,1)\}$$