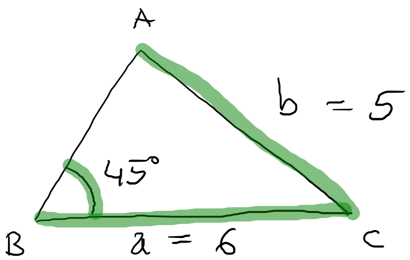
12.04.22

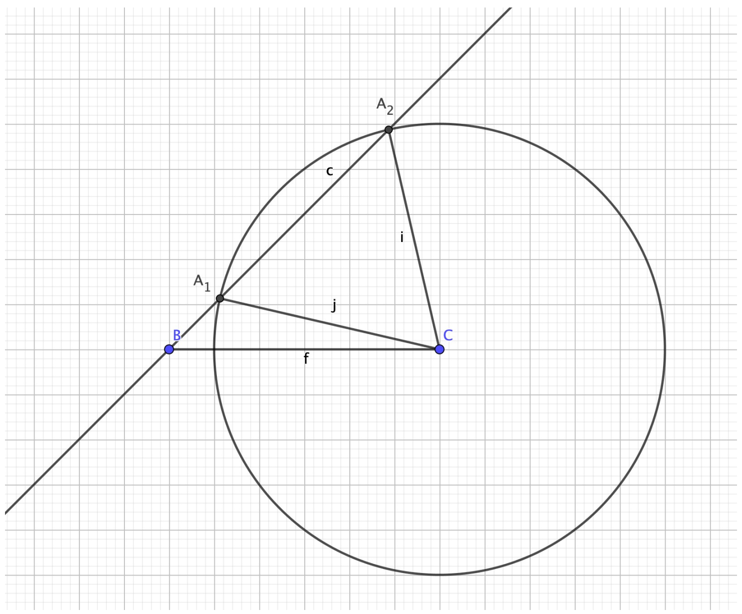
4.4.3 Construire les triangles ABC dont on connaît :

a) $a = 6$ cm $\quad b = 5$ cm $\quad \beta = 45°$

b) $c = 8$ cm $\quad \beta = 30°$ $\quad \gamma = 120°$

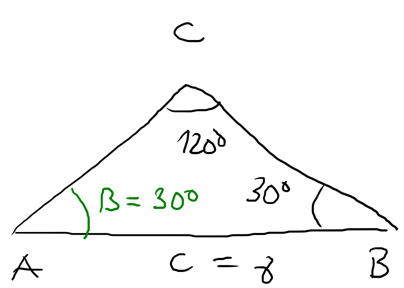
a)

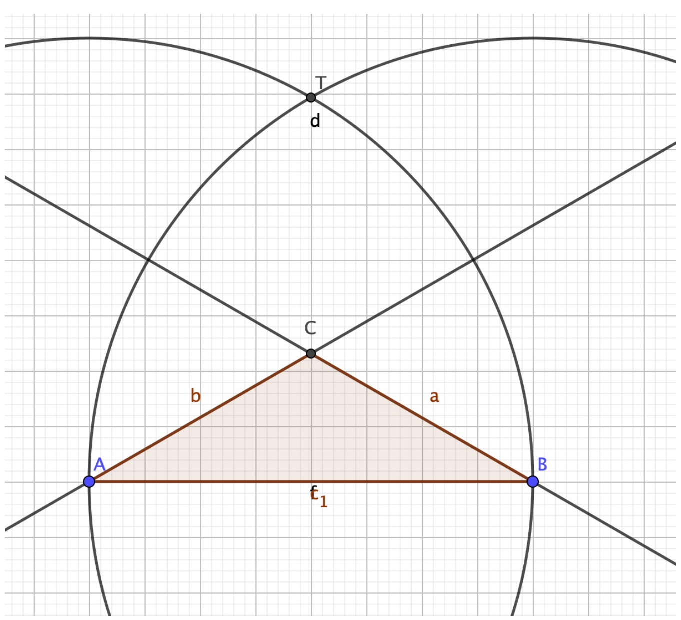




Il y à deux triangles possibles

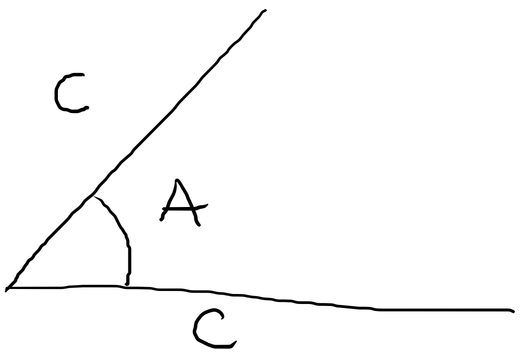
b) $c = 8$ cm $\quad \beta = 30°$ $\quad \gamma = 120°$





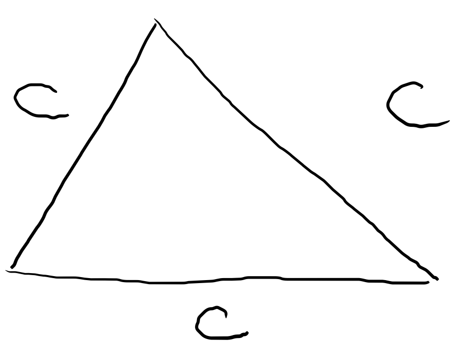
Une seule possibilité

Cas d'isométrie des triangles



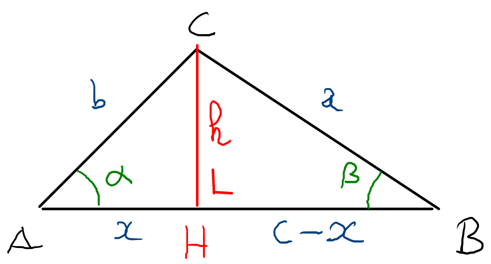
CAC

ACA



CCC avec la somme des deux plus petits plus grand que le troisième

Théorème du cosinus



Soit ABC qcq avec H, pied de la hauteur issue de C sur AB, entre A et B

Posons $x = AH$. Donc $c - x = HB$. Posons $h = CH$

Nous avons $\cos(\alpha) = \dfrac{x}{b}$ $\Rightarrow$ $\boxed{x = \cos(\alpha) \cdot b}$ ΔAHC

$$\begin{cases} h^2 = b^2 - x^2 & \Delta CHA \\ h^2 = a^2 - (c-x)^2 & \Delta CHB \end{cases}$$

donc

$$b^2 - x^2 = a^2 - (c-x)^2$$

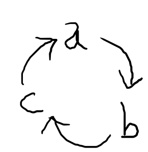
$$b^2 - x^2 = a^2 - (c^2 - 2cx + x^2)$$

$$b^2 - x^2 = a^2 - c^2 + 2cx - x^2$$

$$b^2 + c^2 = a^2 + 2cx$$

$$a^2 = b^2 + c^2 - 2cx$$

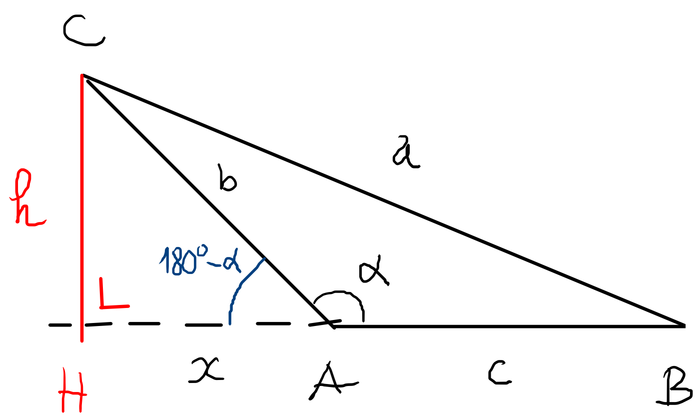
$$\boxed{a^2 = b^2 + c^2 - 2\cos(\alpha)\, b\, c}$$

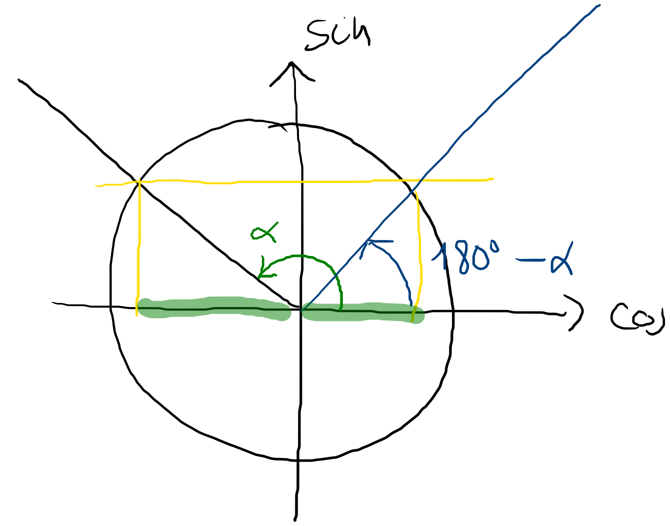


Que se passe-t-il si H tombe à l'extérieur de AB ?

$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos(\alpha) \\ b^2 &= c^2 + a^2 - 2ca\cos(\beta) \\ c^2 &= a^2 + b^2 - 2ab\cos(\gamma) \end{aligned}}$$

Deuxième cas





ΔCHA : $\widehat{CAH} = 180° - \alpha$. On a $\cos(180° - \alpha) = \dfrac{x}{b}$

Donc $x = b \cdot \underbrace{\cos(180° - \alpha)}_{-\cos(\alpha)} = -b\cos(\alpha)$

$$\begin{cases} h^2 = b^2 - x^2 & \Delta CHA \\ h^2 = a^2 - (c+x)^2 & \Delta CHB \end{cases}$$

Donc, nous avons :

$$b^2 - x^2 = a^2 - (c+x)^2$$

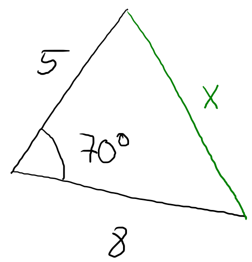
$$b^2 - x^2 = a^2 - c^2 - 2cx - x^2$$

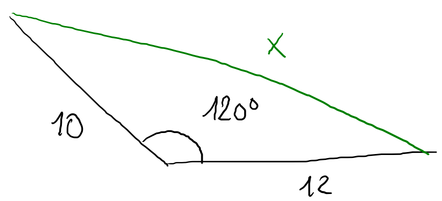
$$a^2 = b^2 + c^2 + 2cx$$

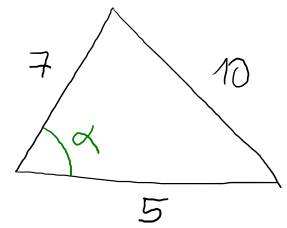
$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

C'est la même relation.

Application

1)


2)


3)


1)
$$x^2 = 5^2 + 8^2 - 2 \cdot 5 \cdot 8 \cdot \cos(70°)$$
$$x^2 \cong 25 + 64 - 27{,}36 \cong 61{,}63$$
$$x \cong 7{,}85$$

2)
$$x^2 = \underbrace{10^2 + 12^2} - 2 \cdot 10 \cdot 12 \cdot \cos(120°)$$
$$x^2 = 244 - 240 \cdot \cos(120°)$$
$$x^2 \cong 244 + 120 = 364$$
$$x \cong 19{,}07$$

3)
$$10^2 = 7^2 + 5^2 - 2 \cdot 7 \cdot 5 \cdot \underline{\cos(\alpha)} \qquad \text{CL}$$
$$10^2 - 7^2 - 5^2 = -70 \cos(\alpha) \qquad \text{CL}$$
$$26 = -70 \cos(\alpha) \qquad \div (-70)$$
$$\cos(\alpha) = \frac{26}{-70} \qquad \text{(éq. trigo)}$$

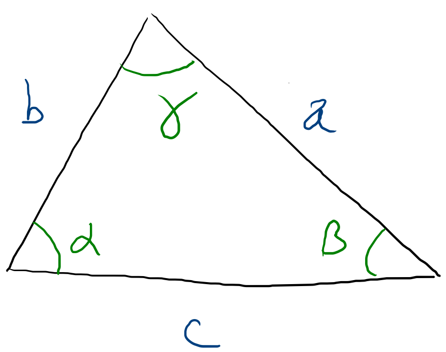
$\boxed{\text{TI}}$ $\alpha \cong$ 111.80°

$$\begin{cases} \alpha_1 \cong 111{,}80° + k \cdot 360° \\ \alpha_2 \cong -111{,}80° + k \cdot 360° \end{cases}$$

Finalement $\alpha \cong 111{,}80°$

4.4.4 Chaque ligne du tableau ci-dessous donne des informations sur un triangle ABC quelconque On sait que l'on a mesuré les longueurs en centimètres et les angles en degrés.

	a	b	c	α	β	γ	$\mathcal{A}$
a)	5	6	7				
b)	5	7		35			
c)	4	9				54	
d)			8	40	80		
e)	6	5					12
f)		4		70			10



Aire

a) Par Nilo ☺

$\alpha = ?$

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$
$$5^2 = 6^2 + 7^2 - 2\cdot6\cdot7\cos(\alpha)$$
$$5^2 - 6^2 - 7^2 = -2\cdot6\cdot7\cdot\cos(\alpha)$$
$$25 - 36 - 49 = -84\cdot\cos(\alpha)$$
$$-60 = -84\cdot\cos(\alpha)$$
$$\cos(\alpha) = \frac{60}{84}$$
$$T_{\alpha} \cong 44{,}41^\circ$$

$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$
$$\underset{36}{6^2} = \overbrace{5^2 + 7^2}^{74} - \overbrace{2\cdot5\cdot7}^{70}\cos(\beta)$$
$$70\cos(\beta) = 74 - 36$$
$$70\cos(\beta) = 38$$
$$\cos(\beta) \cong 0{,}543$$
$$\arccos(0{,}543) \cong 57{,}12$$