

2.5.21 Résoudre les systèmes linéaires ci-dessous :

$$m) \begin{cases} 3x - y + z = 29 \\ x + 3y + 30z = 6 \\ x - y + z = 17 \end{cases}$$

$$m) \begin{cases} 3x - 1y + 1z = 29 \\ 1x + 3y + 30z = 6 \\ 1x - 1y + 1z = 17 \end{cases}$$

$$\begin{array}{l} L_1 \\ L_2 \\ L_3 \end{array} \left(\begin{array}{ccc|c} 3 & -1 & 1 & 29 \\ 1 & 3 & 30 & 6 \\ 1 & -1 & 1 & 17 \end{array} \right) \begin{array}{l} \cup \\ L_1 \leftrightarrow L_2 \\ L_3 \leftarrow L_3 - L_2 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 30 & 6 \\ 3 & -1 & 1 & 29 \\ 0 & -4 & -29 & 11 \end{array} \right) \begin{array}{l} \cup \\ L_2 \leftarrow L_2 - 3L_1 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 30 & 6 \\ 0 & -10 & -89 & 11 \\ 0 & -4 & -29 & 11 \end{array} \right) \begin{array}{l} \cup \\ L_2 \leftarrow -2 \cdot L_2 \\ L_3 \leftarrow 5 \cdot L_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 30 & 6 \\ 0 & 20 & 178 & -22 \\ 0 & -20 & -145 & 55 \end{array} \right) \begin{array}{l} \cup \\ L_3 \leftarrow L_3 + L_2 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 30 & 6 \\ 0 & 20 & 178 & -22 \\ 0 & 0 & 33 & 33 \end{array} \right) \begin{array}{l} \cup \\ L_3 \leftarrow \frac{1}{33} L_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 30 & 6 \\ 0 & 20 & 178 & -22 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$\begin{array}{l} \cup \\ L_2 \leftarrow L_2 - 178L_3 \\ L_1 \leftarrow L_1 - 30L_3 \end{array} \left(\begin{array}{ccc|c} 1 & 3 & 0 & -24 \\ 0 & 20 & 0 & -200 \\ 0 & 0 & 1 & 1 \end{array} \right) \begin{array}{l} \cup \\ L_2 \leftarrow \frac{1}{20} L_2 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 0 & -24 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$\begin{array}{l} \cup \\ L_1 \leftarrow L_1 - 3L_2 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -10 \\ 1 & 0 & 1 & 1 \end{array} \right)$$

$$S = \{ (6; -10; 1) \}$$

$$r) \begin{cases} x - y + z = 0 \\ -x + y + z = 10 \\ x + y - z = 2 \end{cases}$$

$$\begin{array}{l} L_1 \\ L_2 \\ L_3 \end{array} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -1 & 1 & 1 & 10 \\ 1 & 1 & -1 & 2 \end{array} \right) \cup \begin{array}{l} L_2 \leftarrow L_2 + L_1 \\ L_3 \leftarrow L_3 - L_1 \end{array} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 2 & 10 \\ 0 & 2 & -2 & 2 \end{array} \right) \begin{array}{l} L_2 \leftarrow \frac{1}{2} L_2 \\ L_3 \leftarrow \frac{1}{2} L_3 \\ L_2 \leftrightarrow L_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 5 \end{array} \right) \cup \begin{array}{l} L_1 \leftarrow L_1 + L_2 \\ L_2 \leftarrow L_2 + L_3 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 5 \end{array} \right) \quad \mathcal{S} = \left\{ (1; 6; 5) \right\}$$

$$n) \begin{cases} 2x + 3y + 4z = 47 \\ 3x + 5y - 4z = 2 \\ 4x + 7y - 2z = 31 \end{cases}$$

$$-\frac{39}{5}, 11, \frac{37}{5}$$

$$\begin{array}{l} L_1 \\ L_2 \\ L_3 \end{array} \left(\begin{array}{ccc|c} 2 & 3 & 4 & 47 \\ 3 & 5 & -4 & 2 \\ 4 & 7 & -2 & 31 \end{array} \right) \begin{array}{l} L_3 \leftarrow L_3 - 2L_1 \\ L_1 \leftarrow \frac{1}{2} L_1 \end{array} \cup \left(\begin{array}{ccc|c} 1 & 3/2 & 2 & 47/2 \\ 3 & 5 & -4 & 2 \\ 0 & 1 & -10 & -63 \end{array} \right) \cup$$

$$L_2 \leftarrow L_2 - 3L_1 \left(\begin{array}{ccc|c} 1 & 3/2 & 2 & 47/2 \\ 0 & 1/2 & -10 & -137/2 \\ 0 & 1 & -10 & -63 \end{array} \right) \cup \begin{array}{l} L_2 \leftarrow 2 \cdot L_2 \\ L_3 \leftarrow L_3 - L_2 \end{array} \left(\begin{array}{ccc|c} 1 & 3/2 & 2 & 47/2 \\ 0 & 1 & -20 & -137 \\ 0 & 0 & 10 & 74 \end{array} \right)$$

$$\cup \begin{array}{l} L_3 \leftarrow \frac{1}{10} L_3 \\ L_2 \leftarrow L_2 + 20L_3 \\ L_1 \leftarrow L_1 - 2L_3 \end{array} \left(\begin{array}{ccc|c} 1 & 3/2 & 2 & 47/2 \\ 0 & 1 & -20 & -137 \\ 0 & 0 & 1 & 37/5 \end{array} \right) \cup \left(\begin{array}{ccc|c} 1 & 3/2 & 0 & 37/10 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 37/5 \end{array} \right)$$

$$\cup \begin{array}{l} L_1 \leftarrow L_1 - \frac{3}{2} L_2 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -39/5 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 37/5 \end{array} \right) \quad \mathcal{S} = \left\{ \left(-\frac{39}{5}; 11; \frac{37}{5} \right) \right\}$$