Géométrie vectorielle - Série 2

Exercice 1

a) $\vec{BA} = \vec{OA} - \vec{OB} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\vec{AE} = \vec{AO} + \vec{OE} = -\vec{OA} - \vec{EO}$
$= -\vec{OA} - \vec{OB} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

$\vec{EC} = \vec{EB} + \vec{BC} = 2\vec{OB} + \vec{AO}$
$= -\vec{OA} + 2\vec{OB} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$\vec{OD} = \vec{AO} = -\vec{OA} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

b) $\vec{BA} = \vec{DE} = -\vec{ED} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$\vec{AE} = \vec{AD} + \vec{DE} = -2\vec{EF} - \vec{ED}$
$= \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

$\vec{EC} = \vec{EF} + \vec{FC} = \vec{EF} + 2\vec{ED}$
$= \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\vec{OD} = -\vec{FE} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

Exercice 2

$$\vec{u} = \begin{pmatrix} 12 \\ -16 \end{pmatrix} - \begin{pmatrix} -18 \\ 12 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 31 \\ -31 \end{pmatrix}$$

Exercice 3

Comme $\vec{GE} = \vec{CA}$, et que

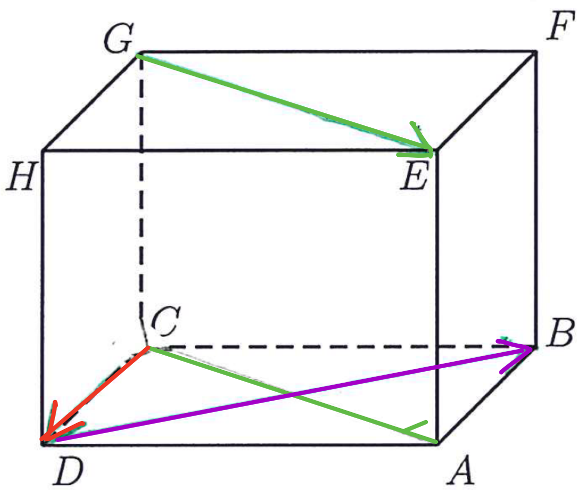
$\vec{CA} = \vec{CD} + \vec{DA} = \vec{CD} + \vec{CB} = \vec{CD} + \vec{DB} - \vec{DC}$

$= \vec{CD} + \vec{DB} + \vec{CD}$

$= 2\vec{CD} + \vec{DB}$

Donc $\vec{GE} = \vec{DB} + 2\vec{CD}$

Les vecteurs sont coplanaires.



Exercice 4

$\vec{OM} = \vec{OA} + \vec{AM} = \begin{pmatrix} -4 \\ 10 \end{pmatrix} + \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} -7 \\ 14 \end{pmatrix} \Rightarrow M(-7 ; 14)$

Problème 5

a) $\vec{u} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - 2\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix}$

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b) $\left|\begin{array}{ccc|cc} 1 & -1 & 1 & 1 & -1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{array}\right| = 0 + 0 + 1 - 0 - 1 - (-1) = 1 \neq 0$

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Les vecteurs $\vec{a}$, $\vec{b}$ et $\vec{c}$ sont linéairement indépendants.

$\mathcal{B}_1$ est donc une base de V_3

c) 1 $\vec{b} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{\mathcal{B}_1}$

d) 1 $\vec{u} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}_{\mathcal{B}_1}$

e) $\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} = \vec{v}$

3 $\begin{cases} \alpha - \beta + \gamma = 10 \\ \alpha + \gamma = 5 \\ \beta + \gamma = -3 \end{cases} \begin{array}{|l} \beta \\ 1 \\ \\ 1 \end{array} \quad \begin{cases} \alpha + 2\gamma = 7 \\ \alpha + \gamma = 5 \\ \beta = -\gamma - 3 \end{cases} \begin{array}{|l} \alpha \\ 1 \\ \cdot(-1) \\ \end{array} \quad \begin{array}{l} \gamma = 2 \\ \alpha = 3 \\ \beta = -5 \end{array} \Rightarrow \vec{v} = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$