07.01.25

2.5.21 Résoudre les systèmes linéaires ci-dessous :

p) $\begin{cases} x + 2y - 3z = 0 \\ 5x - 3y + z = 0 \end{cases}$

1 degré de liberté

1 solution évidente $x=0,\ y=0,\ z=0$

système homogène

$$\begin{array}{c} L_1 \\ L_2 \end{array}\left(\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 5 & -3 & 1 & 0 \end{array}\right) \underset{L_2 \leftarrow L_2 - 5L_1}{\sim} \left(\begin{array}{ccc|c} \boxed{1} & 2 & -3 & 0 \\ 0 & -13 & 16 & 0 \end{array}\right) \underset{L_2 \leftarrow -\frac{1}{13} L_2}{\sim} \left(\begin{array}{ccc|c} \boxed{1} & 2 & -3 & 0 \\ 0 & 1 & -\frac{16}{13} & 0 \end{array}\right)$$

$$\underset{L_1 \leftarrow L_1 - 2L_2}{\sim} \left(\begin{array}{ccc|c} \boxed{1} & 0 & \frac{-7}{13} & \\ 0 & \boxed{1} & -\frac{16}{13} & 0 \end{array}\right)$$

$$-3 - 2\cdot\left(-\frac{16}{13}\right) = -3 + \frac{32}{13} = \frac{-39+32}{13} = \frac{-7}{13}$$

Le système devient : $\begin{cases} x - \frac{7}{13} z = 0 \\ y - \frac{16}{13} z = 0 \end{cases} \Leftrightarrow \begin{cases} x = \frac{7}{13} z \\ y = \frac{16}{13} z \\ z = z \end{cases} \Leftrightarrow \boxed{\begin{cases} x = \frac{7}{13} t \\ y = \frac{16}{13} t \\ z = t \end{cases} \quad t \in \mathbb{R}}$

Une solution $z = 13$, $x = 7$ et $y = 16$

s) $\left\{\begin{array}{rl} x+2\,y+3\,z = & 9 \\ x-y+4\,z = & 15 \\ -x+7\,y-6\,z = & -27 \end{array}\right.$

$$\begin{array}{c} L_1 \\ L_2 \\ L_3 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 1 & -1 & 4 & 15 \\ -1 & 7 & -6 & -27 \end{array}\right) \underset{\substack{L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 + L_1}}{\sim} \left(\begin{array}{ccc|c} \boxed{1} & 2 & 3 & 9 \\ 0 & -3 & 1 & 6 \\ 0 & 9 & -3 & -18 \end{array}\right) \underset{L_3 \leftarrow L_3 + 3L_2}{\sim}$$

$$\left(\begin{array}{ccc|c} \boxed{1} & 2 & 3 & 9 \\ 0 & -3 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right) \underset{L_2 \leftarrow \frac{-1}{3} L_2}{\sim} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & -1/3 & -2 \end{array}\right) \underset{L_1 \leftarrow L_1 - 2L_2}{\sim} \left(\begin{array}{ccc|c} \boxed{1} & 0 & \frac{11}{3} & 13 \\ 0 & \boxed{1} & -\frac{1}{3} & -2 \end{array}\right)$$

$$\left\{\begin{array}{l} x + \frac{11}{3} z = 13 \\ y - \frac{1}{3} z = -2 \\ z = t \end{array}\right. \Leftrightarrow \boxed{\left\{\begin{array}{l} x = \frac{-11}{3} t + 13 \\ y = \frac{1}{3} t - 2 \\ z = t \end{array}\right. , \quad t \in \mathbb{R}}$$

w) $\begin{cases} x + y + 2z = 4 \\ 3x - y + z = 3 \end{cases}$

$$\begin{array}{c} L_1 \\ L_2 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 3 & -1 & 1 & 3 \end{array}\right) \underset{L_2 \leftarrow L_2 - 3L_1}{\sim} \left(\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 4 \\ 0 & -4 & -5 & -9 \end{array}\right) \underset{L_2 \leftarrow -\frac{1}{4} L_2}{\sim}$$

$$\left(\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 4 \\ 0 & 1 & \frac{5}{4} & \frac{9}{4} \end{array}\right) \underset{L_1 \leftarrow L_1 - L_2}{\sim} \left(\begin{array}{ccc|c} \boxed{1} & 0 & -\frac{3}{4} & \frac{7}{4} \\ 0 & \boxed{1} & \frac{5}{4} & \frac{9}{4} \end{array}\right)$$

$$\Rightarrow \begin{cases} x - \frac{3}{4} z = \frac{7}{4} \\ y + \frac{5}{4} z = \frac{9}{4} \\ z = t \end{cases} \Leftrightarrow \begin{cases} x = \frac{3}{4} t + \frac{7}{4} \\ y = -\frac{5}{4} t + \frac{9}{4} \\ z = t \end{cases}$$