08.01.25

2.5.22 Résoudre les systèmes suivants :

a) $\begin{cases} 2\,xy - \ \ 3\,y = 3 \\ y^2 - 4\,xy = -15 \end{cases}$

Ce système n'est pas linéaire

$$\Leftrightarrow \begin{cases} 4xy - 6y = 6 \\ y^2 - 4xy = -15 \end{cases} \Leftrightarrow \begin{cases} 4xy = 6y + 6 \\ y^2 - (6y+6) = -15 \end{cases}$$

$$\Leftrightarrow \begin{cases} 4xy = 6y + 6 \\ y^2 - 6y + 9 = 0 \end{cases} \Leftrightarrow \begin{cases} 4xy = 6y + 6 \\ (y-3)^2 = 0 \end{cases} \Leftrightarrow \begin{cases} 12x = 24 \\ y = 3 \end{cases} \Leftrightarrow \begin{cases} x = 2 \\ y = 3 \end{cases}$$

$$S = \{(2;3)\}$$

a) $$\begin{cases} 2xy - 3y = 3 \\ y^2 - 4xy + 15 = 0 \end{cases} \Leftrightarrow \begin{cases} 2xy = 3y + 3 \\ y^2 - 4xy + 15 = 0 \end{cases} \Bigg| \div (2y)$$ ⚠ $y \neq 0$

$$\begin{cases} \boxed{x} = \dfrac{3y+3}{2y} \\ y^2 - 4\boxed{x}y + 15 = 0 \end{cases} \Leftrightarrow \begin{cases} ① \ x = \dfrac{3y+3}{2y} \\ ② \ y^2 - \overset{2}{\cancel{4}} \dfrac{3y+3}{\cancel{2y}} \cdot \cancel{y} + 15 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \dfrac{3y+3}{2y} \\ y^2 - 2(3y+3) + 15 = 0 \end{cases} \Leftrightarrow \begin{cases} x = \dfrac{3y+3}{2y} \\ y^2 - 6y + 9 = 0 \end{cases} \Leftrightarrow \begin{cases} x = \dfrac{3y+3}{2y} \\ (y-3)^2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 2 \\ y = 3 \end{cases} \qquad S = \{(2,3)\}$$

Que se passe-t-il si $y = 0$?

$$\begin{cases} 0 = 3 \\ \dots \end{cases}$$ impossible !

b) $\begin{cases} \dfrac{x}{y} - \dfrac{y}{x} = \dfrac{5}{6} \\ x + y = 30 \end{cases}$ conditions : $x \neq 0$, $y \neq 0$

$\Leftrightarrow \begin{cases} \dfrac{x^2 - y^2}{xy} = \dfrac{5xy}{6xy} \\ x + y = 30 \end{cases} \quad \Big| \cdot 6xy \ (x \neq 0, y \neq 0)$

$\Leftrightarrow \begin{cases} ① \ 6(x^2 - y^2) = 5xy \\ ② \ y = -x + 30 \end{cases} \quad \Leftrightarrow \begin{cases} ① \ 6x^2 - 6y^2 = 5xy \\ ② \ y = -x + 30 \end{cases}$

Substituons ② dans ① et résolvons cette équation.

① $6x^2 - 6(-x+30)^2 = 5x(-x+30)$

$6x^2 - 6(x^2 - 60x + 900) = -5x^2 + 150x$

$6x^2 - 6x^2 + 360x - 5400 = -5x^2 + 150x$

$5x^2 + 210x - 5400 = 0$

$x^2 + 42x - 1080 = 0$

$\Delta = 42^2 - 4 \cdot 1 \cdot (-1080) = 1764 + 4320 = 6084 = 78^2$

$\left[\begin{array}{l} x_1 = \dfrac{-42+78}{2} = 18 \\ x_2 = \dfrac{-42-78}{2} = -60 \end{array} \right. \quad \overset{②}{\Rightarrow} \left[\begin{array}{l} y_1 = -18 + 30 \\ y_2 = +60 + 30 \end{array} \right. \Rightarrow \left[\begin{array}{l} y_1 = 12 \\ y_2 = 90 \end{array} \right.$

Les solutions : $\begin{cases} x = 18, \ y = 12 \\ \text{ou} \\ x = -60, \ y = 90 \end{cases}$

$S = \{ (18, 12), (-60, 90) \}$

h) $\begin{cases} x + y = 9 \\ x^2 - y^2 = 9 \end{cases}$

$$\Leftrightarrow \begin{cases} x+y = 9 \\ (x+y)(x-y) = 9 \end{cases} \Leftrightarrow \begin{cases} x+y = 9 \\ \cancel{9}^{1}(x-y) = \cancel{9}^{1} \end{cases}$$

$$\Leftrightarrow \begin{cases} x+y = 9 \\ x-y = 1 \end{cases} \left| \begin{array}{c} y \\ \cdot 1 \\ \cdot 1 \end{array} \right| \begin{array}{c} x \\ \cdot 1 \\ \cdot(-1) \end{array} \Leftrightarrow \begin{cases} 2x = 10 \\ 2y = 8 \end{cases} \Leftrightarrow \begin{cases} x = 5 \\ y = 4 \end{cases}$$

$$\Rightarrow S = \{ (5;4) \}$$