

19.12.24

2.5.21 Résoudre les systèmes linéaires ci-dessous :

$$g) \begin{cases} 2x + 3y + 2z = 41 \\ 8x + 5y = 31 \\ 7y = 21 \end{cases}$$

La matrice augmentée du système :

$$\begin{array}{l} L_1 \\ L_2 \\ L_3 \end{array} \left(\begin{array}{ccc|c} 2 & 3 & 2 & 41 \\ 8 & 5 & 0 & 31 \\ 0 & 7 & 0 & 21 \end{array} \right) \begin{array}{l} L_2 \leftrightarrow L_3 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 2 & 3 & 2 & 41 \\ 0 & 7 & 0 & 21 \\ 8 & 5 & 0 & 31 \end{array} \right) \begin{array}{l} L_2 \leftarrow \frac{1}{7} L_2 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 2 & 3 & 2 & 41 \\ 0 & 1 & 0 & 3 \\ 8 & 5 & 0 & 31 \end{array} \right) \begin{array}{l} L_3 \leftarrow L_3 - 4L_2 \\ \\ \end{array} \sim$$

$$\left(\begin{array}{ccc|c} 2 & 3 & 2 & 41 \\ 0 & 1 & 0 & 3 \\ 0 & -7 & -8 & -133 \end{array} \right) \begin{array}{l} L_3 \leftarrow L_3 + 7L_2 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 2 & 3 & 2 & 41 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & -8 & -112 \end{array} \right) \begin{array}{l} L_3 \leftarrow -\frac{1}{8} L_3 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 2 & 3 & 2 & 41 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 14 \end{array} \right)$$

$$\begin{array}{l} L_1 \\ \\ \end{array} \left(\begin{array}{ccc|c} 2 & 0 & 2 & 32 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 14 \end{array} \right) \begin{array}{l} L_1 \leftarrow L_1 - 3L_2 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 2 & 0 & 2 & 32 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 14 \end{array} \right) \begin{array}{l} L_1 \leftarrow L_1 - 2L_3 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 2 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 14 \end{array} \right) \begin{array}{l} L_1 \leftarrow \frac{1}{2} L_1 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} \boxed{1} & 0 & 0 & 2 \\ 0 & \boxed{1} & 0 & 3 \\ 0 & 0 & \boxed{1} & 14 \end{array} \right)$$

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$$\Rightarrow \begin{cases} x = 2 \\ y = 3 \\ z = 14 \end{cases}$$

Ex 2.5.21

k) - m) - r) - n) - q) - p)

$$m) \begin{cases} 3x - y + z = 29 \\ x + 3y + 30z = 6 \\ x - y + z = 17 \end{cases}$$

$$\begin{array}{l} L_1 \\ L_2 \\ L_3 \end{array} \left(\begin{array}{ccc|c} 3 & -1 & 1 & 29 \\ 1 & 3 & 30 & 6 \\ 1 & -1 & 1 & 17 \end{array} \right) \begin{array}{l} L_1 \leftarrow L_1 - 2L_2 \\ L_3 \leftarrow L_3 - L_2 \\ \sim \end{array} \left(\begin{array}{ccc|c} 1 & -7 & -59 & 17 \\ 1 & 3 & 30 & 6 \\ 0 & -4 & -29 & 11 \end{array} \right) \begin{array}{l} L_2 \leftarrow L_2 - L_1 \\ \sim \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -7 & -59 & 17 \\ 0 & 10 & 89 & -11 \\ 0 & -4 & -29 & 11 \end{array} \right) \begin{array}{l} L_2 \leftarrow 2 \cdot L_2 \\ L_3 \leftarrow 5L_3 \\ \sim \end{array} \left(\begin{array}{ccc|c} 1 & -7 & -59 & 17 \\ 0 & 20 & 178 & -22 \\ 0 & -20 & -145 & 55 \end{array} \right) \begin{array}{l} L_3 \leftarrow L_3 + L_2 \\ \sim \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -7 & -59 & 17 \\ 0 & 20 & 178 & -22 \\ 0 & 0 & 33 & 33 \end{array} \right) \begin{array}{l} L_3 \leftarrow \frac{1}{33}L_3 \\ \sim \end{array} \left(\begin{array}{ccc|c} 1 & -7 & -59 & 17 \\ 0 & 20 & 178 & -22 \\ 0 & 0 & 1 & 1 \end{array} \right) \begin{array}{l} L_2 \leftarrow L_2 - 178L_3 \\ \sim \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -7 & -59 & 17 \\ 0 & 20 & 0 & -200 \\ 0 & 0 & 1 & 1 \end{array} \right) \begin{array}{l} L_2 \leftarrow \frac{1}{20}L_2 \\ \sim \end{array} \left(\begin{array}{ccc|c} 1 & -7 & -59 & 17 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 1 \end{array} \right) \begin{array}{l} L_1 \leftarrow L_1 + 59L_3 \\ \sim \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -7 & 0 & 76 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 1 \end{array} \right) \begin{array}{l} L_1 \leftarrow L_1 + 7L_2 \\ \sim \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$\Rightarrow \begin{cases} x = 6 \\ y = -10 \\ z = 1 \end{cases}$$

Mardi 07.01.25

Ex 2.5.21 m) - r) - n) - o)

Ex 2.5.13 d) - f)

Ex 2.5.14 c) - d) - e) - f)