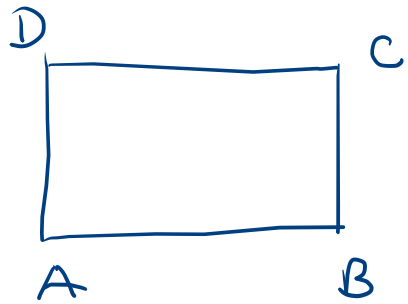


**1.4.13** Montrer que le quadrilatère  $ABCD$  est un rectangle, si  $A(-4; 5; 3)$ ,  $B(-1; 1; 5)$ ,  $C(5; 5; 4)$  et  $D(2; 9; 2)$ .



$$\left. \begin{array}{l} 1) \vec{AB} = \vec{DC} \\ 2) \vec{AB} \perp \vec{BC} \end{array} \right\} \Rightarrow ABCD \text{ est un rectangle}$$

$$1) \vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix} - \begin{pmatrix} -4 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}$$

$$\vec{CD} = \vec{OD} - \vec{OC} = \begin{pmatrix} 2 \\ 9 \\ 2 \end{pmatrix} - \begin{pmatrix} 5 \\ 5 \\ 4 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ -2 \end{pmatrix} \Rightarrow \vec{DC} = \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = \vec{AB}$$

$$2) \vec{AB} \perp \vec{BC} \Leftrightarrow \vec{AB} \cdot \vec{BC} = 3 \cdot 6 + (-4) \cdot 4 + 2 \cdot (-1) = 18 - 16 - 2 = 0$$

$$\vec{BC} = \vec{OC} - \vec{OB} = \begin{pmatrix} 5 \\ 5 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix}$$