2.5 Equations et systèmes d'équations

Formule de Viète

(1) $x^2 + 2x - 3 = 0 \quad \Big| +3$

$\underbrace{x^2 + 2x + 1}_{} = 3 + 1 \quad \Big| +1$

$(x+1)^2 = 4$

$x + 1 = \begin{cases} 2 \\ -2 \end{cases}$

$x + 1 = 2 \Rightarrow x = 1$

$x + 1 = -2 \Rightarrow x = -3$

$S = \{-3; 1\}$

Avec Viète : $x^2 + 2x - 3 = 0$

$a = 1, \quad b = 2, \quad c = -3$

$\Delta = 4 - 4 \cdot 1 \cdot (-3) = 16 = \quad \sqrt{\Delta} = 4$

$x_1 = \frac{-2+4}{2} = 1 \quad ; \quad x_2 = \frac{-2-4}{2} = -3$

(2) $x^2 + 2x - 15 = 0$

$x^2 + 2x + 1 = 15 + 1$

$(x+1)^2 = 16$

$x+1 = 4 \Rightarrow x = 3$

$x+1 = -4 \Rightarrow x = -5$

(3) $x^2 + 2x - 14 = 0$

$x^2 + 2x + 1 = 14 + 1$

$(x+1)^2 = 15$

$x+1 = \sqrt{15} \Rightarrow x = -1 + \sqrt{15}$

$x+1 = -\sqrt{15} \Rightarrow x = -1 - \sqrt{15}$

(4) $x^2 + 6x + 8 = 0$

$x^2 + 6x + 9 = -8 + 9$

$(x + 3)^2 = 1$

$x+3 = 1 \Rightarrow x = -2$

$x+3 = -1 \Rightarrow x = -4$

$x^2 + 6x + 9 = (x+3)^2$

$x^2 - 8x + 16 = (x-4)^2$

(5) $2x^2 - 7x - 4 = 0 \quad | \div 2$

$x^2 - \frac{7}{2}x - 2 = 0$

$x^2 - \frac{7}{2}x + \frac{49}{16} = 2 + \frac{49}{16}$

$\left(x - \frac{7}{4}\right)^2 = \frac{81}{16}$

$x - \frac{7}{4} = \frac{9}{4} \Rightarrow x = \frac{9}{4} + \frac{7}{4} = 4$

$x - \frac{7}{4} = -\frac{9}{4} \Rightarrow x = -\frac{9}{4} + \frac{7}{4} = \frac{-1}{2}$

$a = 2$; $b = -7$; $c = -4$

$\Delta = 49 + 32 = 81$

$x_1 = \frac{7+9}{4} = 4$; $x_2 = \frac{7-9}{4} = \frac{-1}{2}$

Démonstration :

$$ax^2 + bx + c = 0 \quad \Big| \div a, \quad a \neq 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Posons $\Delta = b^2 - 4ac$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{\Delta}{(2a)^2}$$

Trois cas se présentent :

1) Si $\Delta < 0$: aucune solution

2) Si $\Delta = 0$: $\left(x + \frac{b}{2a}\right)^2 = 0 \Rightarrow x = \frac{-b}{2a}$

une solution double

3) Si $\Delta > 0$:

$$x + \frac{b}{2a} = \frac{\sqrt{\Delta}}{2a} \Rightarrow x = \frac{-b}{2a} + \frac{\sqrt{\Delta}}{2a} = \frac{-b + \sqrt{\Delta}}{2a}$$

$$x + \frac{b}{2a} = \frac{-\sqrt{\Delta}}{2a} \Rightarrow x = \frac{-b - \sqrt{\Delta}}{2a}$$

2.5.1 Résoudre les équations ci-dessous :

d) $(x-6)(x+1)+(2x+3)(x-5)=0$

$$\underline{x^2}+\underline{\underline{x}}-\underline{\underline{6x}}-6+\underline{2x^2}-\underline{\underline{10x}}+\underline{\underline{3x}}-15=0 \qquad \text{CL}$$

$$3x^2-12x-21=0 \qquad \text{CL}$$

$$x^2-4x-7=0 \qquad \div 3$$

$a=1,\quad b=-4,\quad c=-7$

$\Delta = 16+28=44$

$$x_1=\frac{4+\sqrt{44}}{2}=\frac{4+2\sqrt{11}}{2}=\frac{\cancel{2}(2+\sqrt{11})}{\cancel{2}}=2+\sqrt{11}$$

$$x_2=\frac{4-\sqrt{44}}{2}=2-\sqrt{11}$$

$$\sqrt{44}=\sqrt{4\cdot 11}=\sqrt{4}\cdot\sqrt{11}=2\sqrt{11}$$