2.4.4 09.11.22

c) $\frac{1}{x} - \frac{x}{x^2-1} - \frac{2x+1}{x-x^3} = \frac{1}{x} - \frac{x}{x^2-1} + \frac{2x+1}{x^3-x}$

- x
- $x^2 - 1 = (x-1)(x+1)$
- $x - x^3 = x(1-x^2) = x(1-x)(1+x)$
  $= -x(x-1)(x+1)$

$x^3 - x = x(x-1)(x+1)$

$$= \frac{(x-1)(x+1) - x^2 + 2x + 1}{x(x-1)(x+1)} = \frac{x^2-1-x^2+2x+1}{x(x-1)(x+1)}$$

$$= \frac{2\cancel{x}}{\cancel{x}(x-1)(x+1)} = \frac{2}{(x-1)(x+1)} = \frac{2}{x^2-1}$$

2.4.3

j) $\frac{13-5x}{6x^2-6}+\frac{3x}{x+1}-\frac{3x-5}{3x-3}$

- $6x^2-6 = 6(x^2-1) = 6(x-1)(x+1)$
- $x+1$
- $3(x-1)$

dc: $6(x-1)(x+1)$

$$= \frac{(13-5x)\cdot 1}{6(x-1)(x+1)} + \frac{3x \cdot 6(x-1)}{6(x-1)(x+1)} - \frac{(3x-5)\cdot 2(x+1)}{6(x-1)(x+1)}$$

$$= \frac{(13-5x) + (18x^2-18x) - (6x^2+6x-10x-10)}{6(x-1)(x+1)}$$

$$= \frac{13-5x+18x^2-18x-6x^2-6x+10x+10}{6(x-1)(x+1)}$$

$$= \frac{12x^2-19x+23}{6(x-1)(x+1)}$$

g) $$\frac{x+2}{x^2+7x+10}-\frac{x-3}{x^2-8x+15}+\frac{x^2-15}{x^2-25}$$

- $x^2+7x+10 = (x+2)(x+5)$
- $x^2-8x+15 = (x-3)(x-5)$
- $x^2-25 = (x-5)(x+5)$

$$= \frac{\cancel{x+2}^{\,1}}{\cancel{(x+2)}_{1}(x+5)} - \frac{\cancel{x-3}^{\,1}}{\cancel{(x-3)}_{1}(x-5)} + \frac{x^2-15}{(x-5)(x+5)}$$

$$= \frac{1}{x+5} - \frac{1}{x-5} + \frac{x^2-15}{(x-5)(x+5)}$$

$$= \frac{x-5-x-5+x^2-15}{(x-5)(x+5)} = \frac{x^2-25}{x^2-25} = 1$$

① Résolution générale de l'équation $ax + b = 0$

$$ax + b = 0 \quad \Big| -b$$
$$ax = -b$$

1^er cas $\quad a = 0$

$$0 = -b$$

- Si $b = 0$ $\qquad S = \mathbb{R}$
- Si $b \neq 0$ $\qquad S = \emptyset$

2^ème cas $\quad a \neq 0$

$$ax = -b \quad \Big| \div a$$
$$x = -\frac{b}{a}$$
$$S = \left\{ -\frac{b}{a} \right\}$$

Résolution de l'équation $ax^2+bx+c=0$, avec $a\neq 0$

Exemples:

1) $x^2+10x+24=0 \quad \Big| -24$

$$\underbrace{x^2+10x+25}=-24+25$$

$$(x+5)^2=1$$

$$\begin{cases} x+5=1 & \Rightarrow x=-4 \\ \text{ou} & \\ x+5=-1 & \Rightarrow x=-6 \end{cases}$$

$$S=\{-6\,;-4\}$$

2) $x^2-6x-40=0$

$$x^2-6x=40$$

$$\underbrace{x^2-6x+9}=40+9$$

$$(x-3)^2=49$$

$$\begin{cases} x-3=7 & \Rightarrow x=10 \\ \text{ou} & \\ x-3=-7 & \Rightarrow x=-4 \end{cases}$$

$$S=\{-4\,;10\}$$

3) $$x^2 + 8x + 3 = 0$$

$$x^2 + 8x + 16 = -3 + 16$$

$$(x+4)^2 = 13$$

$$\left\{\begin{array}{l} x + 4 = \sqrt{13} \quad \Rightarrow \quad x = -4 + \sqrt{13} \\ \text{ou} \\ x + 4 = -\sqrt{13} \quad \Rightarrow \quad x = -4 - \sqrt{13} \end{array}\right.$$

4) $$x^2 + 5x + 8 = 0$$

$$x^2 + 5x = -8$$

$$x^2 + 5x + \frac{25}{4} = -8 + \frac{25}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{-7}{4}$$

$$S = \emptyset$$

5) $$6x^2 + 11x + 3 = 0 \quad \Big| \div 6$$

$$x^2 + \frac{11}{6}x + \frac{1}{2} = 0$$

$$x^2 + \frac{11}{6}x + \frac{121}{144} = -\frac{1}{2} + \frac{121}{144}$$

$$\left(x + \frac{11}{12}\right)^2$$

$$\left(x + \frac{11}{12}\right)^2 = \frac{-72 + 121}{144}$$

$$\left(x + \frac{11}{12}\right)^2 = \frac{49}{144}$$

$$\left\{\begin{array}{l} x + \frac{11}{12} = \frac{7}{12} \quad \Rightarrow \quad x = -\frac{1}{3} \\ \\ x + \frac{11}{12} = -\frac{7}{12} \quad \Rightarrow \quad x = \frac{-18}{12} = -\frac{3}{2} \end{array}\right.$$

$$S = \left\{ -\frac{3}{2} \, ; \, -\frac{1}{3} \right\}$$

$$\boxed{\sqrt{9} = 3}$$

$$\boxed{x^2 = 9}$$

$$x = \pm 3$$