Analyse – Révision 3

Exercice 1

Calculer les limites suivantes.

a) $\lim\limits_{x\to 1} \dfrac{-2\,x^2 - x + 3}{x-1}$

b) $\lim\limits_{x\to -4} \dfrac{x^2 + 4\,x}{-x^2 - 2\,x + 8}$

c) $\lim\limits_{\substack{x\to 1\\<}} \dfrac{x^2 + \text{[redacted]}\,x + 1}{x^2 + x - 2}$

d) $\lim\limits_{\substack{x\to 1\\>}} \dfrac{x^2 + \text{[redacted]}\,x + 1}{x^2 + x - 2}$

e) $\lim\limits_{x\to 0} \left(\dfrac{1}{x^2+x} - \dfrac{1}{x}\right)$

f) $\lim\limits_{x\to \text{[redacted]}} \left(\dfrac{1}{x-1} - \dfrac{2}{x^2-1}\right)$ 1

a) $\lim\limits_{x\to 1} \dfrac{-2x^2 - x + 3}{x-1} \underset{\text{“}\frac{0}{0}\text{”}}{\overset{\text{Ind}}{=}} \lim\limits_{x\to 1} \dfrac{-(2x^2+x-3)}{x-1} =$

$\lim\limits_{x\to 1} \dfrac{-\cancel{(x-1)}(2x+3)}{\cancel{x-1}_{\,1}} = \lim\limits_{x\to 1} \dfrac{-(2x+3)}{1} = -5$

b) $\lim\limits_{x\to -4} \dfrac{x^2+4x}{-x^2-2x+8} \underset{\text{“}\frac{0}{0}\text{”}}{\overset{\text{Ind}}{=}} \lim\limits_{x\to -4} \dfrac{x(x+4)}{-(x^2+2x-8)} =$

$\lim\limits_{x\to -4} \dfrac{x\cancel{(x+4)}}{-\cancel{(x+4)}(x-2)} = \lim\limits_{x\to -4} \dfrac{x}{-(x-2)} = \dfrac{-4}{-(-6)} = -\dfrac{2}{3}$

c) $\lim\limits_{\substack{x\to 1\\<}} \dfrac{x^2+\quad x+1}{x^2+x-2} \underset{\text{“}\frac{3}{0}\text{”}}{=} \infty$

$f(x) = \dfrac{x^2+\quad x+1}{x^2+x-2}$

$x^2+x-2 = (x+2)(x-1)$

x		-2		1	
$x^2+\quad x+1$	$+$		$+$		$+$
x^2+x-2	$+$	0	$-$	0	$+$
$f(x)$	$+$	$\Vert$	$-$	$\Vert$	$+$

$$\lim_{\substack{x \to 1 \\ <}} f(x) = -\infty$$

d) $$\lim_{\substack{x \to 1 \\ >}} \frac{x^2+x+1}{x^2+x-2} \underset{\text{"}\frac{3}{0}\text{"}}{=} +\infty$$

e) $$\lim_{x \to 0} \left(\frac{1}{x^2+x} - \frac{1}{x} \right) \underset{\text{"}\infty-\infty\text{"}}{\overset{\text{Ind}}{=}} \lim_{x \to 0} \left(\frac{1}{x(x+1)} - \frac{x+1}{x(x+1)} \right) =$$

$$\lim_{x \to 0} \frac{1-x-1}{x(x+1)} = \lim_{x \to 0} \frac{-\cancel{x}^{\,-1}}{\cancel{x}(x+1)} = \lim_{x \to 0} \frac{-1}{x+1} = -1$$

f) $$\lim_{x \to 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1} \right) \underset{\text{"}\infty-\infty\text{"}}{\overset{\text{Ind}}{=}} \lim_{x \to 1} \left(\frac{1}{x-1} - \frac{2}{(x-1)(x+1)} \right)$$

$$= \lim_{x \to 1} \frac{x+1-2}{(x-1)(x+1)} \underset{\text{"}\frac{0}{0}\text{"}}{\overset{\text{Ind}}{=}} \lim_{x \to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)} = \lim_{x \to 1} \frac{1}{x+1}$$

$$= \frac{1}{2}$$

Exercice 2

Calculer les limites suivantes.

a) $\lim\limits_{x\to 2} \dfrac{x^2-4}{\sqrt{x}-\sqrt{2}}$

b) $\lim\limits_{x\to 0} \dfrac{1-\sqrt{1-x}}{x}$

c) $\lim\limits_{x\to 1} \dfrac{\sqrt{x+3}-2}{x-1}$

d) $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{x-1}$

e) $\lim\limits_{x\to 2} \dfrac{\sqrt{x+2}-\sqrt{3\,x-2}}{x-2}$

f) $\lim\limits_{x\to 4} \dfrac{\sqrt{3\,x+4}-4}{x-4}$

a) $\lim\limits_{x\to 2} \dfrac{(x-2)(x+2)}{\sqrt{x}-\sqrt{2}} \underset{\text{“}\frac{0}{0}\text{”}}{\overset{\text{ind}}{=}} \lim\limits_{x\to 2} \dfrac{(x-2)(x+2)}{\sqrt{x}-\sqrt{2}} \cdot \dfrac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}} =$

$\lim\limits_{x\to 2} \dfrac{(x-2)(x+2)(\sqrt{x}+\sqrt{2})}{(\sqrt{x}-\sqrt{2})(\sqrt{x}+\sqrt{2})} = \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x+2)(\sqrt{x}+\sqrt{2})}{\underset{1}{\cancel{x-2}}}$

$= \lim\limits_{x\to 2} \dfrac{(x+2)(\sqrt{x}+\sqrt{2})}{1} = 4\cdot \underbrace{(\sqrt{2}+\sqrt{2})}_{2\sqrt{2}} = 8\sqrt{2}$

b) $\lim\limits_{x\to 0} \dfrac{1-\sqrt{1-x}}{x} \underset{\text{“}\frac{0}{0}\text{”}}{\overset{\text{ind}}{=}} \lim\limits_{x\to 0} \dfrac{1-\sqrt{1-x}}{x} \cdot \dfrac{1+\sqrt{1-x}}{1+\sqrt{1-x}}$

$= \lim\limits_{x\to 0} \dfrac{1-(1-x)}{x(1+\sqrt{1-x})} = \lim\limits_{x\to 0} \dfrac{\overset{1}{\cancel{x}}}{\cancel{x}(1+\sqrt{1-x})} = \lim\limits_{x\to 0} \dfrac{1}{1+\sqrt{1-x}}$

$= \dfrac{1}{2}$

c) $\lim\limits_{x\to 1} \dfrac{\sqrt{x+3}-2}{x-1} \underset{\text{“}\frac{0}{0}\text{”}}{\overset{\text{ind}}{=}} \lim\limits_{x\to 1} \dfrac{\sqrt{x+3}-2}{x-1} \cdot \dfrac{\sqrt{x+3}+2}{\sqrt{x+3}+2}$

$$= \lim_{x\to 1} \frac{x+3-4}{(x-1)(\sqrt{x+3}+2)} = \lim_{x\to 1} \frac{\cancel{x-1}^{\,1}}{\cancel{(x-1)}(\sqrt{x+3}+2)} =$$

$$\lim_{x\to 1} \frac{1}{\sqrt{x+3}+2} = \frac{1}{4}$$

d) $$\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} \underset{\text{"}\frac{0}{0}\text{"}}{\overset{\text{ind}}{=}} \lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} =$$

$$\lim_{x\to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(\sqrt{x}+1)} = \lim_{x\to 1} \frac{1}{\sqrt{x}+1} = \frac{1}{2}$$

e) $$\lim_{x\to 2} \frac{\sqrt{x+2}-\sqrt{3x-2}}{x-2} \underset{\text{"}\frac{0}{0}\text{"}}{\overset{\text{ind}}{=}}$$

$$\lim_{x\to 2} \frac{\sqrt{x+2}-\sqrt{3x-2}}{x-2} \cdot \frac{\sqrt{x+2}+\sqrt{3x-2}}{\sqrt{x+2}+\sqrt{3x-2}} = \lim_{x\to 2} \frac{(x+2)-(3x-2)}{(x-2)(\sqrt{\ } + \sqrt{\ })}$$

$$= \lim_{x\to 2} \frac{-2x+4}{(x-2)(\sqrt{x+2}+\sqrt{3x-2})} = \lim_{x\to 2} \frac{-2\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{x+2}+\sqrt{3x-2})}$$

$$= \frac{-2}{(2+2)} = \frac{-2}{4} = \frac{-1}{2}$$

$$= \lim_{x\to 4} \frac{\sqrt{3x+4}-4}{x-4} \underset{\text{"}\frac{0}{0}\text{"}}{\overset{\text{ind}}{=}} \lim_{x\to 4} \frac{\sqrt{3x+4}-4}{x-4} \cdot \frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4} =$$

$$\lim_{x\to 4} \frac{3x+4-16}{(x-4)(\sqrt{3x+4}+4)} = \lim_{x\to 4} \frac{3x-12}{(x-4)(\sqrt{3x+4}+4)} =$$

$$\lim_{x\to 4} \frac{3\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{3x+4}+4)} = \frac{3}{8}$$

Exercice 3

Calculer les limites suivantes.

a) $\displaystyle\lim_{x\to+\infty} \frac{36\,x^2-15\,x+200}{(3\,x-2)^2}$

c) $\displaystyle\lim_{x\to-\infty} \frac{4\,x^2-x+3}{2\,x-1}$

b) $\displaystyle\lim_{x\to-\infty} \frac{1000\,x^3-50\,x^2+7}{x^4+x^2+x+1}$

d) $\displaystyle\lim_{x\to-\infty} \frac{5\,x^2+4\,x-1}{x+2}$

a) $$\lim_{x\to+\infty} \frac{36x^2-15x+200}{9x^2-12x+4} = \lim_{x\to+\infty} \frac{36x^2\left(1\overbrace{-\frac{15}{36x}+\frac{200}{36x^2}}^{\to 0}\right)}{9x^2\left(1\underbrace{-\frac{12}{9x^2}+\frac{4}{9x^2}}_{\to 0}\right)}$$

$$\lim_{x\to+\infty} \frac{36x^2}{9x^2} = 4$$

b) $$\lim_{x\to-\infty} \frac{1000x^3\left(1\overbrace{-\frac{50}{1000x^3}+\frac{7}{1000x^3}}^{\to 0}\right)}{x^4\left(1\underbrace{+\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4}}_{\to 0}\right)} = \lim_{x\to-\infty} \frac{1000}{x} = 0$$

c) $$\lim_{x\to-\infty} \frac{4x^2-x+3}{2x-1} = \infty$$

Tableau des signes : $\displaystyle\frac{4x^2-x+3}{2x-1} = \frac{(4x+3)(x-1)}{2x-1} = f(x)$

$ED(f) = \mathbb{R} - \left\{\frac{1}{2}\right\}$

x		$-\frac{3}{4}$		$\frac{1}{2}$		1	
$f(x)$	$-$	0	$+$	$\Vert$	$-$	0	$+$

$$\lim_{x\to-\infty} f(x) = -\infty$$

Exercice 4

Soit $f(x) = \dfrac{x^2+x-6}{|x-2|}$.

Calculer $\lim\limits_{\substack{x\to 2 \\ <}} f(x)$, $\lim\limits_{\substack{x\to 2 \\ >}} f(x)$ et $\lim\limits_{x\to 2} f(x)$.

$$f(x) = \frac{x^2+x-6}{|x-2|} = \begin{cases} \dfrac{(x+3)\cancel{(x-2)}}{\cancel{x-2}} = x+3 & x > 2 \\[2ex] \dfrac{(x+3)(x-2)}{-(x-2)} = -(x+3) & x < 2 \end{cases}$$

Donc $\lim\limits_{\substack{x\to 2 \\ <}} f(x) = -(2+3) = -5$

$\lim\limits_{\substack{x\to 2 \\ >}} f(x) = 2+3 = 5$

Ainsi $\lim\limits_{x\to 2} f(x)$ n'existe pas

<u>Ex 3</u>

d) $\lim\limits_{x\to -\infty} \dfrac{5x^2+4x-1}{x+2} = \infty$

$f(x) = \dfrac{(5x-1)(x+1)}{x+2}$ $\quad ED(f) = \mathbb{R} - \{-2\}$

x		-2		-1		$1/5$	
$f(x)$	$-$	$\parallel$	$+$		$-$	0	$+$

$\lim\limits_{x\to -\infty} f(x) = -\infty$