

1.2.1

$$\boxed{a^x = a^y \Leftrightarrow x = y}$$

avec  $a \in \mathbb{R}_+^*$

$$\begin{aligned} \text{a) } 7^{x+6} &= 7^{3x+4} \Rightarrow x+6 = 3x+4 \\ -2x &= -2 \\ x &= 1 \end{aligned}$$

$$S_a = \{1\}$$

$$\begin{aligned} \text{b) } 6^{7-x} &= 6^{2x+1} \Rightarrow 7-x = 2x+1 \\ -3x &= -6 \\ x &= 2 \end{aligned}$$

$$S_b = \{2\}$$

$$\begin{aligned} \text{c) } 3^{2x+3} &= 3^{x^2} \Rightarrow 2x+3 = x^2 \\ x^2 - 2x - 3 &= 0 \\ (x-3)(x+1) &= 0 \\ \downarrow \quad \downarrow \\ x &= 3 \quad x = -1 \end{aligned}$$

$$S_c = \{-1, 3\}$$

$$d) 9^{x^2} = 3^{3x+2}$$

$$(3^2)^{x^2} = 3^{3x+2}$$

$$3^{2x^2} = 3^{3x+2} \Rightarrow$$

$$2x^2 = 3x + 2$$

$$2x^2 - 3x - 2 = 0$$

$$(2x+1)(x-2) = 0$$

$$\downarrow$$

$$x = -\frac{1}{2}$$

$$\downarrow$$

$$x = 2$$

$$S_d = \left\{ -\frac{1}{2}; 2 \right\}$$

$$e) 2^{-100x} = 0,5^{x-4}$$

$$\boxed{0,5 = \frac{1}{2} = 2^{-1}}$$

$$2^{-100x} = (2^{-1})^{x-4}$$

$$2^{-100x} = 2^{-x+4}$$

$$\Rightarrow -100x = -x + 4$$

$$-99x = 4$$

$$x = -\frac{4}{99}$$

$$S_e = \left\{ -\frac{4}{99} \right\}$$

$$f) \left( \frac{1}{4} \right)^{6-x} = 4$$

$$\frac{1}{4} = 4^{-1}$$

$$4^{-6+x} = 4^1 \Rightarrow$$

$$-6 + x = 1$$

$$x = 7$$

$$S_f = \{ 7 \}$$

$$g) \quad 27^{x-1} = 9^{2x-3}$$

$$27 = 3^3; \quad 9 = 3^2$$

$$3^{3(x-1)} = 3^{2(2x-3)} \Rightarrow 3x-3 = 4x-6$$

$$S_g = \{3\}$$

$$x = 3$$

$$h) \quad 2^x \cdot 4^x = -5$$

Comme  $2^x \cdot 4^x > 0$ , c'est impossible

$$i) \quad (5^{x-2})^4 = \underbrace{5^3}_{125} \cdot 5^{5x-3}$$

$$5^{4x-8} = 5^{5x} \Rightarrow 4x-8 = 5x$$

$$S_i = \{-8\}$$

$$x = -8$$

$$j) \quad (3^{x-1})^3 = 3^2 \cdot 3^{x-2}$$

$$3^{3x-3} = 3^x \Rightarrow 3x-3 = x$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$$S_j = \left\{ \frac{3}{2} \right\}$$

$$K) \underbrace{3^{4x+2}} - 36 \cdot 3^{2x+1} = -243$$

$$\left(3^{2x+1}\right)^2 - 36 \cdot 3^{2x+1} + 243 = 0$$

Posons  $y = 3^{2x+1}$   $y > 0$

$$y^2 - 36y + 243 = 0$$

$$\Delta = 36^2 - 4 \cdot 243 = 324 = 18^2$$

$$y = \frac{36 \pm 18}{2} \Rightarrow \begin{cases} y_1 = 9 \\ y_2 = 27 \end{cases}$$

Determinons  $x$ :

$$\textcircled{1} \quad 3^{2x+1} = 9$$

$$3^{2x+1} = 3^2 \Rightarrow 2x+1 = 2$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$\textcircled{2} \quad 3^{2x+1} = 27$$

$$3^{2x+1} = 3^3 \Rightarrow 2x+1 = 3$$

$$2x = 2$$

$$x = 1$$

$$S_K = \left\{ \frac{1}{2}, 1 \right\}$$

$$1) \quad 5 \cdot 5^{4x-7} - 120 \cdot 5^{2x-3} = 625$$

$$5^1 \cdot 5^{4x-6-1} - 120 \cdot 5^{2x-3} - 625 = 0$$

$$5^1 \cdot 5^{-1} \cdot 5^{2(2x-3)} - 120 \cdot 5^{2x-3} - 625 = 0$$

$$\left(5^{2x-3}\right)^2 - 120 \cdot 5^{2x-3} - 625 = 0$$

$$y^2 - 120 \cdot y - 625 = 0$$

$$\Delta = 120^2 - 4 \cdot (-625) = 16900 = 130^2$$

$$y = \frac{120 \pm 130}{2} \Rightarrow \begin{cases} y_1 = -5 & \text{ne convient pas!} \\ y_2 = 125 \end{cases}$$

$$② \quad 5^{2x-3} = 125$$

$$5^{2x-3} = 5^3 \Rightarrow 2x-3 = 3$$

$$2x = 6$$

$$x = 3$$

$$S_1 = \{3\}$$