2.7.12

a) $f(x) = \underbrace{(x+1)}_{u} \cdot \underbrace{(x-3)}_{v}$

$(u\cdot v)' = u'v + u\cdot v'$

$f'(x) = 1\cdot(x-3) + (x+1)\cdot 1 = \underline{2x-2}$

b) $f'(x) = 1\cdot(x^2+5) + x\cdot(2x) = x^2+5+2x^2 = \underline{3x^2+5}$

c) $f(x) = \underbrace{(7x^2-4x+3)}_{u}\underbrace{(5-2x)}_{v}$

$u' = 14x-4$; $v' = -2$

$f'(x) = (14x-4)(5-2x) + (7x^2-4x+3)\cdot(-2)$

$= 70x - 20 - 28x^2 + 8x - 14x^2 + 8x - 6$

$= \underline{-42x^2 + 86x - 26}$

d) $f(x) = (2x-1)(2-2x)(1+x) = (4x - 4x^2 - 2 + 2x)(1+x)$

$= (-4x^2+6x-2)(1+x)$

$f'(x) = (-8x+6)(1+x) + (-4x^2+6x-2)\cdot 1$

$= -8x - 8x^2 + 6 + 6x - 4x^2 + 6x - 2$

$= \underline{-12x^2 + 4x + 4}$

e) $u = 4-3x$; $u' = -3$

$v = 2x-1$; $v' = 2$

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

$f'(x) = \frac{-3(2x-1) - (4-3x)\cdot 2}{(2x-1)^2} = \frac{-6x+3-8+6x}{(2x-1)^2}$

$= \underline{\frac{-5}{(2x-1)^2}}$

f) $f'(x) = \dfrac{1 \cdot (3-x) - (x-2) \cdot (-1)}{(3-x)^2} = \dfrac{3-x+x-2}{(3-x)^2} = \dfrac{1}{(3-x)^2}$

g) $u = 5 \quad ; \; u' = 0$

$v = 2x^2 - 1 \quad ; \; v' = 4x$

$$f'(x) = \frac{0 - 5 \cdot 4x}{(2x^2-1)^2} = \frac{-20x}{(2x^2-1)^2}$$

h) $u = x^3 - 10x^2 \quad ; \; u' = 3x^2 - 20x$

$v = 1 - x \quad ; \; v' = -1$

$$f'(x) = \frac{(3x^2-20x)(1-x) - (x^3-10x^2) \cdot (-1)}{(1-x)^2}$$

$$= \frac{3x^2 - 20x - 3x^3 + 20x^2 + x^3 - 10x^2}{(1-x)^2} = \frac{-2x^3 + 13x^2 - 20x}{(1-x)^2}$$

i) $u = 8x^2 - 8x + 3 \; ; \; u' = 16x - 8$

$v = 4x^2 - 1 \quad ; \; v' = 8x$

$$f'(x) = \frac{(16x-8)(4x^2-1) - (8x^2-8x+3) \cdot 8x}{(4x^2-1)^2}$$

$$= \frac{64x^3 - 16x - 32x^2 + 8 - 64x^3 + 64x^2 - 24x}{(4x^2-1)^2}$$

$$= \frac{32x^2 - 40x + 8}{(4x^2-1)^2}$$

j) $u = x^3 \quad ; \quad u' = 3x^2$

$v = x+1 \quad ; \quad v' = 1$

$$f'(x) = \frac{3x^2(x+1) - x^3 \cdot 1}{(x+1)^2} = \frac{3x^3 + 3x^2 - x^3}{(x+1)^2} = \frac{2x^3 + 3x^2}{(x+1)^2}$$

k) $f(x) = 1 + \frac{1}{x} - \frac{2}{x^2} = \frac{x^2 + x - 2}{x2}$

$u = x^2 + x - 2 \quad ; \quad u' = 2x + 1$

$v = x^2 \quad ; \quad v' = 2x$

$$f'(x) = \frac{(2x+1) \cdot x^2 - (x^2 + x - 2) \cdot 2x}{x^4} = \frac{2x^3 + x^2 - 2x^3 - 2x^2 + 4x}{x^4}$$

$$= \frac{-x^2 + 4x}{x^4} = \frac{\cancel{x}^{1}(-x+4)}{\cancel{x^4}_{x^3}} = \frac{-x+4}{x^3}$$

l) $f'(x) = \frac{3x^2 \cdot 3x - (x^3 - 4) \cdot 3}{(3x)^2} + 1 = \frac{9x^3 - 3x^3 + 12}{9x^2} + 1$

$$= \frac{6x^3 + 12}{9x^2} + 1 = \frac{\cancel{3}(2x^3 + 4)}{\cancel{9}_{3}x^2} + 1 = \frac{2x^3 + 4}{3x^2} + 1$$

m) $f(x) = \frac{x(x+5)}{x(x+1)} = \frac{x+5}{x+1}$

$$f'(x) = \frac{1 \cdot (x+1) - (x+5) \cdot 1}{(x+1)^2} = \frac{-4}{(x+1)^2}$$

n) $f(x) = x^2 + \frac{2}{x}$

$$f'(x) = 2x - \frac{2}{x^2} = \frac{2x^3 - 2}{x^2}$$

o) $f(x) = \frac{2x}{3} + \frac{\cancelto{1}{3}(x^2-1)}{\cancelto{1}{3}}$

$$f'(x) = \frac{2}{3} + 2x$$