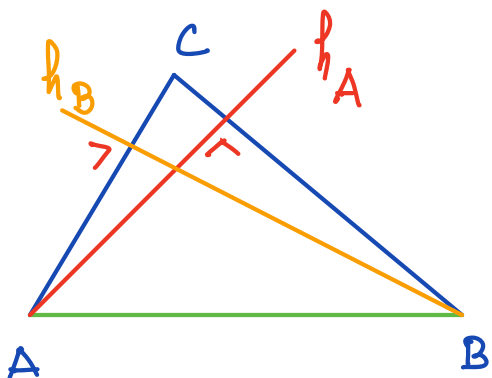


3.1.24 D'un triangle ABC , on donne l'équation du côté $AB : 5x - 3y + 2 = 0$, celle de la hauteur issue de $A : h_A : 4x - 3y - 1 = 0$ ainsi que celle de la hauteur issue de $B : h_B : 7x + 2y - 22 = 0$. Calculer les coordonnées du point C .



$$(AB): 5x - 3y + 2 = 0$$

$$(h_A): 4x - 3y - 1 = 0$$

$$(h_B): 7x + 2y - 22 = 0$$

Déterminons les sommets A et B . Le sommet C est l'intersection de $AC \perp h_B$ et $BC \perp h_A$.

1) Sommet A:

$$\begin{cases} 5x - 3y = -2 \\ 4x - 3y = -1 \end{cases} \begin{array}{l} \cdot 1 \\ \cdot (-1) \end{array} \begin{array}{l} | \\ | \end{array} \begin{array}{l} y \\ x \end{array} \begin{array}{l} | \\ | \end{array} \begin{array}{l} \cdot 4 \\ \cdot (-5) \end{array}$$

$$\Leftrightarrow \begin{cases} x = -1 \\ 3y = -3 \end{cases} \Leftrightarrow \begin{cases} x = -1 \\ y = -1 \end{cases} \Rightarrow A(-1; -1)$$

2) Droite AC: $AC \perp h_B$

$$(AC): 2x - 7y + c = 0 ; \text{ par } A: -2 + 7 + c = 0 \Rightarrow c = -5$$

$$(AC): 2x - 7y - 5 = 0$$

3) Sommet B:

$$\begin{cases} 5x - 3y = -2 \\ 7x + 2y = 22 \end{cases} \begin{array}{l} \cdot 2 \\ \cdot 3 \end{array} \begin{array}{l} | \\ | \end{array} \begin{array}{l} y \\ x \end{array} \begin{array}{l} | \\ | \end{array} \begin{array}{l} \cdot 7 \\ \cdot (-5) \end{array}$$

$$\Leftrightarrow \begin{cases} 31x = 62 \\ -31y = -124 \end{cases} \Leftrightarrow \begin{cases} x = 2 \\ y = 4 \end{cases} \Rightarrow B(2; 4)$$

4) Droite BC: $BC \perp h_A$

$$(BC): 3x + 4y + c = 0; \text{ par B: } 6 + 16 + c = 0 \Rightarrow c = -22$$

$$(BC): 3x + 4y - 22 = 0$$

5) Sommet C:

$$\begin{array}{l} (AC): \begin{cases} 2x - 7y - 5 = 0 \\ 3x + 4y - 22 = 0 \end{cases} \\ (BC): \begin{cases} 2x - 7y - 5 = 0 \\ 3x + 4y - 22 = 0 \end{cases} \end{array} \quad \begin{array}{c|c} y & x \\ \hline \cdot 4 & \cdot 3 \\ \hline \cdot 7 & \cdot (-2) \end{array}$$

$$\Leftrightarrow \begin{cases} 29x - 17y = 0 \\ -29y + 29 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 6 \\ y = 1 \end{cases}$$

$$\Rightarrow C(6; 1)$$