Problème 1

a) $y = (4x^3 - x^2 + 5x - 1)^4$

b) $y = \dfrac{(x-5)^3}{(2x+7)^4}$

c) $y = \dfrac{\sqrt{9x-4}}{(3x+8)^3}$

d) $y = (4x+6)^4 (2x-5)^3$

a) $y' = 4(4x^3 - x^2 + 5x - 1)^3 (12x^2 - 2x + 5)$

b) $u = (x-5)^3 \Rightarrow u' = 3(x-5)^2$

$v = (2x+7)^4 \Rightarrow v' = 8(2x+7)^3$

$$y' = \frac{3(x-5)^2(2x+7)^4 - (x-5)^3 \cdot 8(2x+7)^3}{(2x+7)^8} = \frac{(x-5)^2(2x+7)^3\left[3(2x+7) - 8(x-5)\right]}{(2x+7)^8}$$

$$= \frac{(x-5)^2(-2x+61)}{(2x+7)^5}$$

c) $y = (9x-4)^{1/2}(3x+8)^{-3}$

(i) $y' = \frac{1}{2} \cdot 9 (9x-4)^{-1/2}(3x+8)^{-3} + (9x-4)^{1/2} \cdot (-9)(3x+8)^{-4}$

$$= \frac{9}{2} \frac{1}{\sqrt{9x-4}\ (3x+8)^3} - 9\sqrt{9x-4}\ \frac{1}{(3x+8)^4} = \frac{9\left[\frac{1}{2}(3x+8) - (9x-4)\right]}{\sqrt{9x-4}\ (3x+8)^4}$$

$$= \frac{9}{2} \frac{3x+8-18x+8}{\sqrt{9x-4}\ (3x+8)^4} = \frac{9}{2} \frac{-15x+16}{\sqrt{9x-4}\ (3x+8)^4}$$

(ii) ou $u = \sqrt{9x-4}$ $\quad u' = \dfrac{9}{2\sqrt{9x-4}}$

$v = (3x+8)^3$ $\quad v' = 9(3x+8)^2$

$$y' = \frac{\frac{9}{2\sqrt{9x-4}} \cdot (3x+8)^3 - \sqrt{9x-4} \cdot 9(3x+8)^2}{(3x+8)^6} = \frac{\frac{9(3x+8)^2}{2\sqrt{9x-4}} - 9\sqrt{9x-4}}{(3x+8)^4} = \dots$$

d) $y' = 16(4x+6)^3(2x-5)^3 + (4x+6)^4 \cdot 6(2x-5)^2$

$= 2(4x+6)^3(2x-5)^2\left[8(2x-5) + 3(4x+6)\right]$

$= 2(4x+6)^3(2x-5)^2(28x-22)$

$= 2 \cdot 2^3(2x+3)^3(2x-5)^2 \cdot 2(14x-11) = 32(2x+3)^3(2x-5)^2(14x-11)$

Problème 2

$f'(x) = \dfrac{1}{2\sqrt{x+3}}$ $\qquad y = \frac{1}{2}x - \frac{3}{2}$ $\qquad ED(f') =]-3; +\infty[$

$ED(f) = [-3; +\infty[$

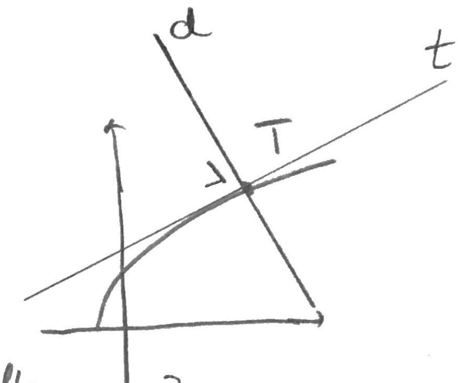
$f'(x) = \dfrac{1}{2\sqrt{x+3}} = \dfrac{1}{2} \iff \sqrt{x+3} = 1$

$x = -2$

Point $T(-2;1)$ $\qquad y = \frac{1}{2}x + h \Rightarrow 1 = -1 + h \Rightarrow h = 2$

$$\boxed{y = \frac{1}{2}x + 2}$$

Problème 3



$f(x) = \sqrt{6x+1}$ $\qquad T(4;5)$

$f'(x) = \dfrac{6}{2\sqrt{6x+1}} = \dfrac{3}{\sqrt{6x+1}}$

pente de la tangente t au point T : $f'(4) = \frac{3}{5}$

pente de la normale d : $-\frac{5}{3}$

droite cherchée : $y = -\frac{5}{3}x + h$, par T : $5 = \frac{-5}{3} \cdot 4 + h$

$h = \frac{35}{3}$

$$\boxed{y = \frac{-5}{3}x + \frac{35}{3}}$$

Problème 4

$f'(x) = 4\left(1 - \frac{x}{2}\right)^3 \cdot \left(-\frac{1}{2}\right) = -2\left(1 - \frac{x}{2}\right)^3 = -2\left(\frac{x-2}{-2}\right)^3 = \frac{(x-2)^3}{4}$

a) en x_0 : $f'(4) = 2$ $\qquad$ en $T(4;1)$

tangente $y = 2x - 7$

b) en x_1 : $f'(3) = \frac{1}{4}$ $\qquad$ en $S(3, \frac{1}{16})$

tangente $y = \frac{1}{4}x - \frac{11}{16}$