Propriétés (suite et fin)

⑧ $\boxed{\log}$ $\boxed{\ln}$

On aimerait calculer le $\log_a$, pour a quelconque ($a>0$, $a \neq 1$).

$$\log_a(x) = \log_a\left(b^{\log_b(x)}\right) \underset{⑦}{=} \log_b(x) \cdot \log_a(b)$$

$$\boxed{⑦ \ \log_a(x^r) = r \log_a(x)}$$

Donc $$\boxed{⑧ \ \log_b(x) = \frac{\log_a(x)}{\log_a(b)}}$$

Par exemple :

$$\ln(100) \overset{\boxed{TI}}{\cong} 4.605170185988091$$

$$\ln(100) = \frac{\log_{10}(100)}{\log_{10}(e)} \cong \frac{2}{0.434294481903252} \cong 4.605170185988092$$

$$\log_{27}(42) = \frac{\ln(42)}{\ln(27)} \cong 1{,}134 \quad \Leftrightarrow \quad 27^{1{,}134} \cong 42$$

04.10.23

4.2.3 Sachant que $\log(2) = 0.3010$ et $\log(3) = 0.4771$, calculer sans la calculatrice :

a) $\log(6)$ b) $\log(16)$ c) $\log(\sqrt{2})$ d) $\log(0,5)$ e) $\log(36)$ f) $\log\left(\frac{8}{27}\right)$

a) $\log(6) = \log(2 \cdot 3) = \log(2) + \log(3)$

$= 0,3010 + 0.4771 = 0,7781$

b) $\log(16) = \log(2^4) = 4 \cdot \log(2) = 1,2040$

c) $\log(\sqrt{2}) = \log\left(2^{\frac{1}{2}}\right) = \frac{1}{2} \log(2) = 0,1505$

d) $\log(0.5) = \log\left(\frac{1}{2}\right) = \log(2^{-1}) = -\log(2) = -0,3010$

e) $\log(36) = \log(4 \cdot 9) = \log(4) + \log(9) = 2\log(2) + 2\log(3)$

$= 1,5562$

f) $\log\left(\frac{8}{27}\right) = \log(8) - \log(27) = 3\log(2) - 3\log(3)$

$= 3\left(\log(2) - \log(3)\right) = -0,5283$

4.2.4 Simplifier les expressions ci-dessous sans utiliser la machine :

a) $\log(16) + 2\log(3) - 2\log(2) - \dfrac{1}{2}\log(9)$

b) $\log(15) + 3\log(10) - \log(30) - \log(5)$

c) $4\log(5) + \log\left(\dfrac{1}{5}\right) - 3\log(3) + \dfrac{1}{3}\log(27)$

d) $\dfrac{\log(20) + \log(100) - \log(2)}{\log(5'000) - \log(5) + \log(0,1)}$

a) $4\log(2) + 2\log(3) - 2\log(2) - \log(3)$

$= \log(2)(4-2) + \log(3)(2-1)$

$= 2\log(2) + \log(3) = \log(12)$

4.2.5 Résoudre les équations ci-dessous :

a) $x = \log_2(32)$ b) $2^x = 100$ c) $\log_x(256) = 4$ d) $\log_2(x) = 4$

e) $10^x = 5$ f) $e^{2x-1} = 27$ g) $\log_x(1'000) = 3$ h) $12^x = -49$

$a = b$

$\Updownarrow$

$\log(a) = \log(b)$

b) $2^x = 100$

$x = \log_2(100)$

$x = \dfrac{\log(100)}{\log(2)}$

$x = \dfrac{2}{\log(2)}$

$2^x = 100$

$\log(2^x) = \log(100)$

$x \cdot \log(2) = 2$

$x = \dfrac{2}{\log(2)}$

4.2.6 Résoudre les équations ci-dessous :

a) $\log_{11}(x+1) = \log_{11}(7)$

b) $\log_6(2x-3) = \log_6(12) - \log_6(3)$

c) $\log(x) - \log(x+1) = 3\log(4)$

d) $2\log_3(x) = 3\log_3(5)$

e) $\ln(x) + \ln(x-2) = 0,5\ln(9)$

f) $\log_8(x+4) = 1 - \log_8(x-3)$

c) $\log(x) - \log(x+1) = 3\log(4)$ (**)

$$\log(x) = \log(x+1) + 3\log(4)$$

$$\log(x) = \log(x+1) + \log(64)$$

$$\log(x) = \log\left(64 \cdot (x+1)\right)$$

$$\Rightarrow \text{(*)} \quad x = 64(x+1)$$

$$x = 64x + 64$$

$$63x = -64$$

$$\boxed{x = \frac{-64}{63}}$$ est une solution de (*)

<u>Preuve</u> : Comme $\log\left(\frac{-64}{63}\right)$ n'existe pas, donc l'équation (**) n'a pas de solution