07.02.24

g bis) $f(x) = (4x+1)^4 \cdot (2x^3+1)^3$

$u = (4x+1)^4$; $u' = 4(4x+1)^3 \cdot 4 = 16(4x+1)^3$

$v = (2x^3+1)^3$; $v' = 3(2x^3+1)^2 \cdot 6x^2 = 18x^2(2x^3+1)^2$

$$f'(x) = 16(4x+1)^3 \cdot (2x^3+1)^3 + (4x+1)^4 \cdot 18x^2(2x^3+1)^2$$

$$= 2(4x+1)^3(2x^3+1)^2\left[8(2x^3+1) + 9x^2(4x+1)\right]$$

$$= \underline{2(4x+1)^3(2x^3+1)^2(52x^3+9x^2+8)}$$

$$[u \cdot v \cdot w]' = u'vw + u \cdot v'w + u \cdot v \cdot w'$$

$$= (uv)'w + uv \cdot w' = [u'v + uv']w + uvw'$$

2.9.10

e) $f(x) = x^2(5x+2)^3 = 2x(5x+2)^3 + x^2 \cdot 15(5x+2)^2$

$u = x^2$; $u' = 2x$

$V = (5x+2)^3$; $V' = 3(5x+2)^2 \cdot 5 = 15(5x+2)^2$

$(uv)' = u'V + uV'$

$= x(5x+2)^2 \left[2(5x+2) + x \cdot 15 \right]$

$= \underline{\underline{x(5x+2)^2(25x+4)}}$

1) $f(x) = \dfrac{x(x-3)^2}{(x-2)^2}$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

① $u = x(x-3)^2$

$\boxed{\begin{array}{ll} u_1 = x & ; \quad u_1' = 1 \\ v_1 = (x-3)^2 & ; \quad v_1' = 2(x-3) \end{array}}$

$$u' = 1\cdot(x-3)^2 + x \cdot 2(x-3)$$
$$= (x-3)\left[(x-3) + 2x\right]$$
$$= (x-3)(3x-3)$$

$v = (x-2)^2$ $\qquad v' = 2(x-2)\cdot 1$

$$f'(x) = \frac{(x-3)(3x-3)(x-2)^2 - x(x-3)^2\cdot 2(x-2)}{(x-2)^4}$$

$$= \frac{(x-3)\cancel{(x-2)}^{1}\left[(3x-3)(x-2) - x(x-3)\cdot 2\right]}{(x-2)^{\cancel{4}\,3}}$$

$$= \frac{(x-3)\left[3x^2 - 6x - 3x + 6 - 2x^2 + 6x\right]}{(x-2)^3}$$

$$= \frac{(x-3)(x^2 - 3x + 6)}{(x-2)^3}$$

1) $f(x) = \dfrac{x(x-3)^2}{(x-2)^2}$

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- $\boxed{(\sqrt{x})' = \dfrac{1}{2\sqrt{x}}}$; $\boxed{(\sqrt{u})' = \dfrac{u'}{2\sqrt{u}}}$

- $(\sqrt{u})' = \left(u^{\frac{1}{2}}\right)' = \dfrac{1}{2}\underbrace{u^{\frac{1}{2}-1}}_{u^{-\frac{1}{2}} = \frac{1}{u^{\frac{1}{2}}} = \frac{1}{\sqrt{u}}} \cdot u' = \dfrac{1}{2}\dfrac{u'}{\sqrt{u}}$

- $\left(\sqrt[3]{u^2}\right)' = \left(u^{\frac{2}{3}}\right)' = \dfrac{2}{3}u^{-\frac{1}{3}} \cdot u' = \dfrac{2}{3}\dfrac{u'}{\sqrt[3]{u}}$

2.9.11

$$(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$$

$$(u^n)' = n\, u^{n-1} \cdot u'$$

c) $f(x) = \sqrt[7]{x^4} = x^{\frac{4}{7}}$

$$f'(x) = \frac{4}{7} x^{-\frac{3}{7}} = \frac{4}{7} \frac{1}{\sqrt[7]{x^3}}$$

d) $f(x) = \sqrt{8x^2 - 5x + 3}$

$$f(x) = (8x^2 - 5x + 3)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} (8x^2 - 5x + 3)^{-\frac{1}{2}} \cdot (16x - 5)$$

$$= \frac{1}{2} \frac{16x - 5}{\sqrt{8x^2 - 5x + 3}}$$

i) $f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$$f'(x) = -\frac{1}{2}\, x^{-\frac{3}{2}} = -\frac{1}{2}\, \frac{1}{\sqrt{x^3}}$$

$(x^n)' = n \cdot x^{n-1} \quad (n \in \mathbb{R})$	$(u^n)' = n \cdot u^{n-1} \cdot u' \quad (n \in \mathbb{R})$
$(\sqrt{x})' = \dfrac{1}{2\sqrt{x}}$	$(\sqrt{u})' = \dfrac{u'}{2\sqrt{u}}$
$(\mathrm{e}^x)' = \mathrm{e}^x$	$(\mathrm{e}^u)' = \mathrm{e}^u \cdot u'$
$\big(\ln(x)\big)' = \dfrac{1}{x} \qquad \big(\ln(\lvert x\rvert)\big)' = \dfrac{1}{x}$	$\big(\ln(u)\big)' = \dfrac{u'}{u} \qquad \big(\ln(\lvert u\rvert)\big)' = \dfrac{u'}{u}$
$\big(\sin(x)\big)' = \cos(x)$	$\big(\sin(u)\big)' = \cos(u) \cdot u'$
$\big(\cos(x)\big)' = -\sin(x)$	$\big(\cos(u)\big)' = -\sin(u) \cdot u'$
$\big(\tan(x)\big)' = 1 + \tan^2(x) = \dfrac{1}{\cos^2(x)}$	$\big(\tan(u)\big)' = \big(1 + \tan^2(u)\big) \cdot u' = \dfrac{u'}{\cos^2(u)}$

f) $f(x) = \sqrt{(4x^2 - 2x)^3}$

$$\boxed{(\sqrt{u})' = \frac{u'}{2\sqrt{u}}} \qquad \boxed{\sqrt{a^3} = a\sqrt{a}} \qquad \frac{a}{\sqrt{a}} = \sqrt{a}$$

$$\sqrt{a^3} = a\sqrt{a}$$

$$f'(x) = \frac{\overset{3}{\cancel{6}}(4x-1)(4x^2-2x)^2}{\underset{1}{\cancel{2}}\sqrt{(4x^2-2x)^3}} = \frac{3(4x-1)(4x^2-2x)^{\overset{1}{\cancel{2}}}}{\cancel{(4x^2-2x)}\sqrt{4x^2-2x}} = \frac{3(4x-1)(4x^2-2x)}{\sqrt{4x^2-2x}}$$

$$= 3(4x-1)\sqrt{4x^2-2x}$$

$$u = (4x^2 - 2x)^3$$

$$u' = 3(4x^2-2x)^2 \cdot (8x-2) = 6(4x-1)(4x^2-2x)^2$$