3.3.1 08.05.24

j) $x^2 + y^2 + y = 0$

k) $80x^2 + 80y^2 - 120x + 80y + 17 = 0$

j) $x^2 + y^2 + y + \frac{1}{4} = 0 + \frac{1}{4}$

$x^2 + (y + \frac{1}{2})^2 = \frac{1}{4}$ $\quad C(0; -\frac{1}{2})$ $\quad r = \frac{1}{2}$

k) $x^2 - \frac{12\cancel{0}}{8\cancel{0}}x + y^2 + y = -\frac{17}{80}$

$x^2 - \frac{3}{2}x + \frac{9}{16} + y^2 + y + \frac{1}{4} = -\frac{17}{80} + \frac{9}{16} + \frac{1}{4}$

$(x - \frac{3}{4})^2 + (y + \frac{1}{2})^2 = \frac{-17 + 45 + 20}{80}$

$(x - \frac{3}{4})^2 + (y + \frac{1}{2})^2 = \frac{48}{80}$

$(x - \frac{3}{4})^2 + (y + \frac{1}{2})^2 = \frac{3}{5}$ $\quad C(\frac{3}{4}; -\frac{1}{2})$ $\quad r = \sqrt{\frac{3}{5}}$

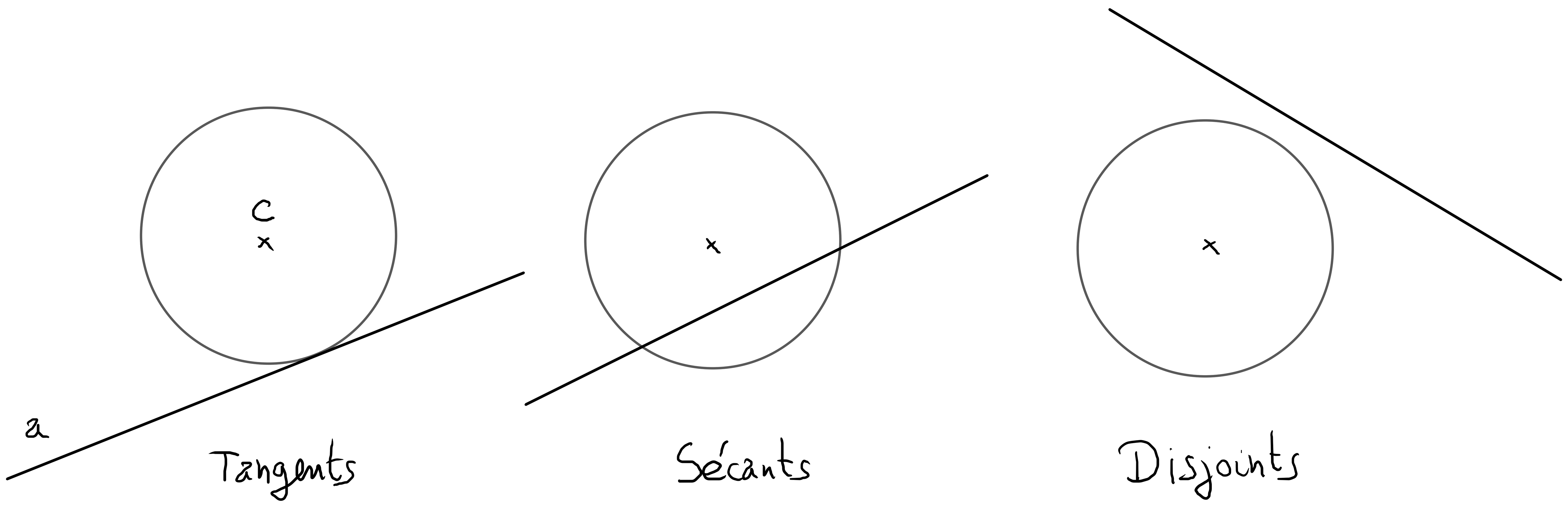
$\sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}} \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$

3.3.3 Déterminer la position relative des deux objets suivants :

a) la droite $y = 2x - 3$ et le cercle $x^2 + y^2 - 3x + 2y = 3$;

b) la droite $x - 2y - 1 = 0$ et le cercle $x^2 + y^2 - 8x + 2y + 12 = 0$;

c) la droite $y = x + 10$ et le cercle $x^2 + y^2 = 1$.



a) $(a) : 2x - y - 3 = 0$

$(\gamma) : \left(x - \frac{3}{2}\right)^2 + (y+1)^2 = \frac{25}{4}$ $\qquad C\left(\frac{3}{2} ; -1\right) \qquad r = \frac{5}{2}$

$$\delta(C, a) = \frac{\left|2 \cdot \frac{3}{2} - (-1) - 3\right|}{\sqrt{5}}$$

$$\delta(P; d) = \frac{|a \cdot p_1 + b \cdot p_2 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{1}{\sqrt{5}} < \frac{5}{2} = r \Rightarrow \delta(C, a) < r \Rightarrow a \text{ et } \gamma \text{ sécants}$$

b) $x^2 + y^2 - 8x + 2y + 12 = 0$; $\qquad x - 2y - 1 = 0$

$(x - 4)^2 + (y + 1)^2 = 5 \qquad C(4 ; -1) \qquad r = \sqrt{5}$

$$\delta(C ; a) = \frac{|4 - 2 \cdot (-1) - 1|}{\sqrt{5}} = \frac{5}{\sqrt{5}} = r \Rightarrow \gamma \text{ et } a \text{ tangents}$$

c) $C(0;0)$, $r = 1 \qquad x - y + 10 = 0$

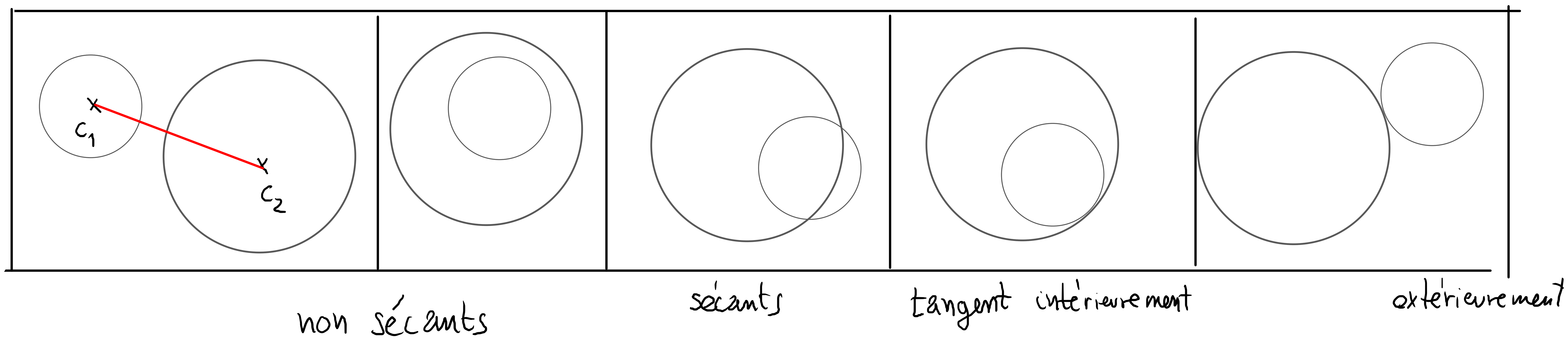
$$\delta(C ; a) = \frac{|0 - 0 + 10|}{\sqrt{2}} = \frac{10}{\sqrt{2}} > 1 \quad \text{disjoints}$$

3.3.4 Déterminer la position relative des cercles

$$\gamma_1 : x^2 + y^2 - 16x - 20y + 115 = 0 \quad \text{et} \quad \gamma_2 : x^2 + y^2 + 8x - 10y + 5 = 0$$

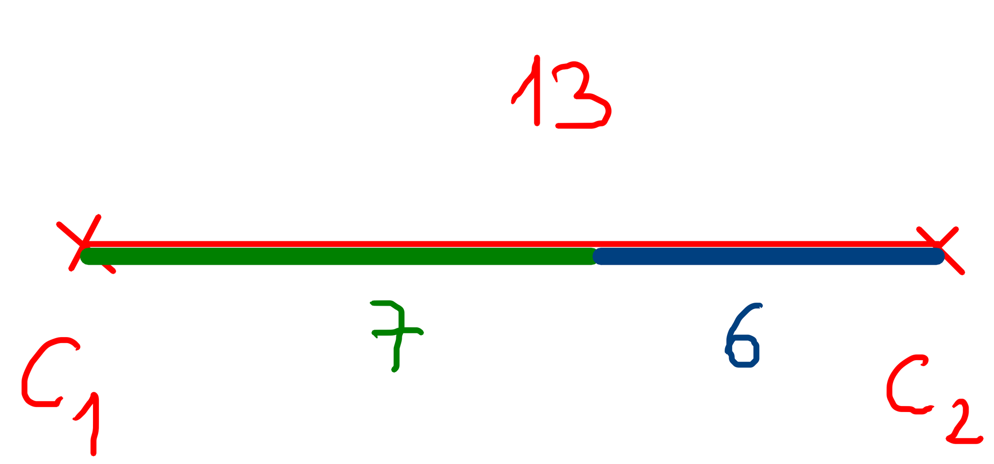
$(\gamma_1) : (x-8)^2 + (y-10)^2 = 49$ $\qquad C_1(8;10)$, $r_1 = 7$

$(\gamma_2) : (x+4)^2 + (y-5)^2 = 36$ $\qquad C_2(-4;5)$, $r_2 = 6$



Distance entre les deux centres :

$$\overrightarrow{C_1C_2} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} - \begin{pmatrix} 8 \\ 10 \end{pmatrix} = \begin{pmatrix} -12 \\ -5 \end{pmatrix} \quad ; \quad \|\overrightarrow{C_1C_2}\| = \sqrt{12^2 + 5^2} = 13$$



$r_1 + r_2 = 7 + 6 = 13 = \|\overrightarrow{C_1C_2}\|$

tangents extérieurement.

Mercredi 15.05.24 : Finir → 3.3.8 (yc)