

10.01.24

2.6.16 Calculer les limites suivantes :

$$a) \lim_{x \rightarrow \infty} \frac{2x - 4}{-3x + 1} = \frac{2}{-3}$$

$$c) \lim_{x \rightarrow \infty} \frac{x^2 + 4x + 29}{x^2 - 2x + 4} = 1$$

$$e) \lim_{x \rightarrow \infty} \frac{(x+1)^7(2x+3)^4}{(2x+1)^3(x-98)^8}$$

$$g) \lim_{x \rightarrow +\infty} \left(\frac{2x - x^3}{3x + 1} + x - 1 \right)$$

$$b) \lim_{x \rightarrow -\infty} \frac{-3x^2 + 1}{x + 2}$$

$$d) \lim_{x \rightarrow \infty} \frac{(3x+4)(x-1)}{(2x+7)(1-5x)}$$

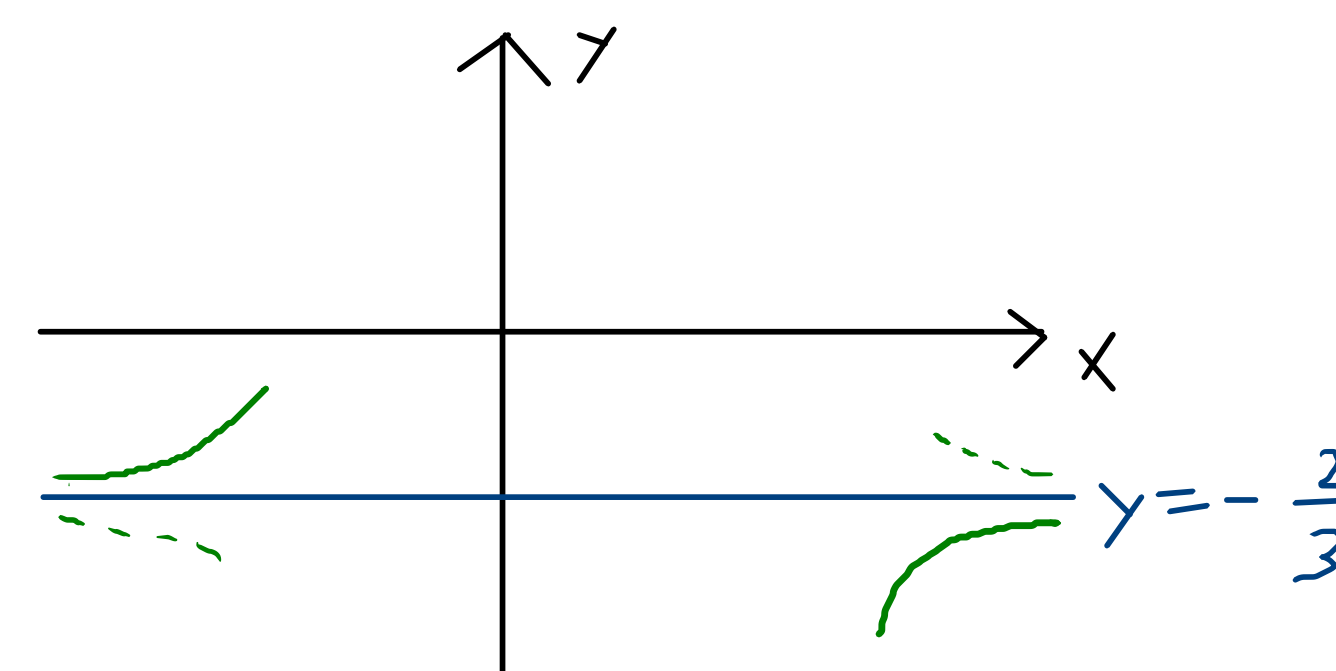
$$f) \lim_{x \rightarrow \infty} \left(\frac{2x^2 - 1}{x - 1} + 1 - 2x \right)$$

$$h) \lim_{x \rightarrow \infty} \left(\frac{1 + 5x - 3x^2}{x - 2} + 3x + 1 \right)$$

$$a) f(x) = \frac{2x - 4}{-3x + 1}$$

Quel est le comportement de cette fonction lorsque $x \rightarrow \infty$?

$$\lim_{x \rightarrow \infty} \frac{2x - 4}{-3x + 1} \stackrel{\text{Ind}}{=} \lim_{x \rightarrow \infty} \frac{\cancel{x} \left(2 - \frac{4}{x} \right)}{\cancel{x} \left(-3 + \frac{1}{x} \right)} = -\frac{2}{3}$$



$$b) \lim_{x \rightarrow -\infty} \frac{-3x^2 + 1}{x + 2} = \lim_{x \rightarrow -\infty} \frac{-3x^2}{x} = \lim_{x \rightarrow -\infty} -3x = +\infty$$

Généralisation

$n, m \in \mathbb{N}^*$, $a_0, \dots, a_n$ et $b_0, \dots, b_m$ des nombres réels, avec $a_n \neq 0$, $b_m \neq 0$.

$$\lim_{x \rightarrow \infty} \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_0} = \begin{cases} 0 & n < m \\ \frac{a_n}{b_m} & n = m \\ \infty & n > m \end{cases}$$

$$e) \lim_{x \rightarrow \infty} \frac{(x+1)^7(2x+3)^4}{(2x+1)^3(x-98)^8} = \lim_{x \rightarrow \infty} \frac{(x^7 + \dots)(16x^4 + \dots)}{(8x^3 + \dots)(x^8 + \dots)}$$

$$= \lim_{x \rightarrow \infty} \frac{16x^{11} + \dots}{8x^{11} + \dots} = 2$$

$$f) \lim_{x \rightarrow \infty} \left(\underbrace{\frac{2x^2 - 1}{x - 1}}_{\text{"}\infty\text{"}} + \underbrace{1 - 2x}_{-\infty} \right) = \lim_{x \rightarrow \infty} \frac{2x^2 - 1 + (1 - 2x)(x - 1)}{x - 1}$$

$$= \lim_{x \rightarrow \infty} \frac{\cancel{2x^2} - 1 + x - 1 - \cancel{2x^2} + 2x}{x - 1} = \lim_{x \rightarrow \infty} \frac{3x - 2}{x - 1} = 3$$

$$g) \lim_{x \rightarrow +\infty} \left(\frac{2x - x^3}{3x + 1} + x - 1 \right) = \lim_{x \rightarrow +\infty} \frac{2x - x^3 + (x - 1)(3x + 1)}{3x + 1}$$

"-∞ + ∞"

$$= \lim_{x \rightarrow +\infty} \frac{2x - x^3 + 3x^2 + x - 3x - 1}{3x + 1} = \lim_{x \rightarrow +\infty} \frac{-x^3 + 3x^2 - 1}{3x + 1}$$

$$= \lim_{x \rightarrow +\infty} \frac{-x^3}{3x} = \lim_{x \rightarrow +\infty} -x^2 = -\infty$$

2.6.17 Calculer, si elles existent, $\lim_{x \rightarrow -\infty}$ et $\lim_{x \rightarrow +\infty}$ pour les fonctions f suivantes :

a) $f(x) = \sqrt{x^2 + x + 1} - x$

b) $f(x) = \frac{\sqrt{4x^2 - 4x + 3}}{x + 1}$

2) Recherchons son ED.

$$f(x) \text{ existe} \Leftrightarrow x^2 + x + 1 \geq 0$$

$$\Delta = -3 \Rightarrow x^2 + x + 1 > 0$$

Ainsi $ED(f) = \mathbb{R}$. On peut ainsi calculer $\lim_{x \rightarrow -\infty} f(x)$ et $\lim_{x \rightarrow +\infty} f(x)$.

$$\textcircled{1} \lim_{x \rightarrow -\infty} (\sqrt{x^2 + x + 1} - x) = +\infty - (-\infty) = +\infty + \infty = +\infty$$

$$\textcircled{2} \lim_{x \rightarrow +\infty} (\sqrt{x^2 + x + 1} - x) \stackrel{\text{Ind}}{=} \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + x + 1} - x)(\sqrt{x^2 + x + 1} + x)}{\sqrt{x^2 + x + 1} + x}$$

$\begin{matrix} +\infty - (+\infty) \\ +\infty - \infty \end{matrix}$

$$= \lim_{x \rightarrow +\infty} \frac{x^2 + x + 1 - x^2}{\sqrt{x^2 + x + 1} + x} = \lim_{x \rightarrow +\infty} \frac{x + 1}{\sqrt{x^2 + x + 1} + x} \stackrel{\text{Ind}}{=}$$

$\begin{matrix} +\infty \\ +\infty \end{matrix}$

$$= \lim_{x \rightarrow +\infty} \frac{x + 1}{\sqrt{x^2 \left(1 + \frac{1}{x} + \frac{1}{x^2}\right)} + x} = \lim_{x \rightarrow +\infty} \frac{x \left(1 + \frac{1}{x}\right)}{\cancel{|x|} \sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} + x}$$

$$= \lim_{x \rightarrow +\infty} \frac{\cancel{x} \left(1 + \frac{1}{x}\right)}{\cancel{x} \left[\sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} + 1\right]} = \frac{1}{1 + 1} = \frac{1}{2}$$