3.3.12 Déterminer les équations des cercles tangents aux droites

$$y = 7x - 5 \quad \text{et} \quad x + y + 13 = 0$$

l'un des points de contact étant $T(1;2)$.

16.05.24

3.3.13

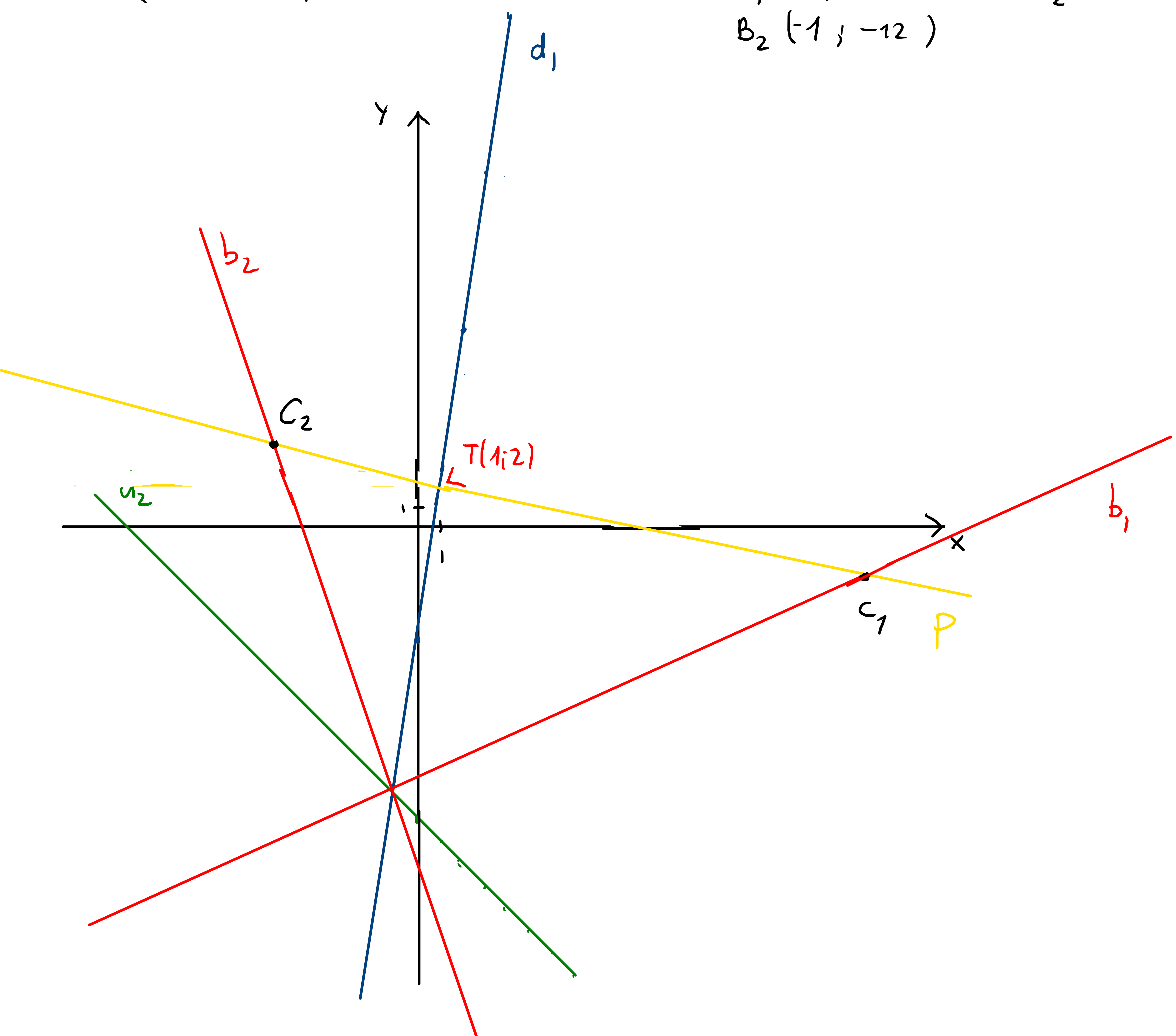
3.3.15

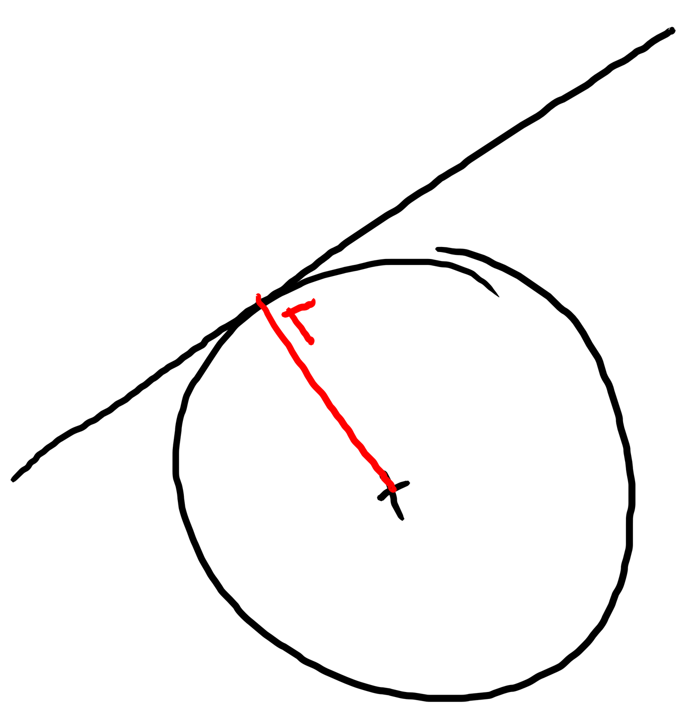
(d_1) : $y = 7x - 5$ $\quad A_1(0; -5)$ $\quad m_1 = 7$

$A_2(1; 2)$

(d_2) : $y = -x - 13$ $\quad B_1(0; -13)$ $\quad m_2 = -1$

$B_2(-1; -12)$





① Les bissectrices

$(b_1) : x - 3y - 35 = 0$ $\quad$ $(b_2) : 3x + y + 15 = 0$

$m_1 = \frac{1}{3}$ $\quad$ $m_2 = -3$ $\quad$ $m_1 \cdot m_2 = -1$

② Perpendiculaire à d_1 par T

$T \in d_1$

$(p) : x + 7y - 15 = 0$ $\quad$ $(d_1) : 7x - y - 5 = 0$

③ Centre des cercles

$$\begin{array}{l} (b_1) : \\ (p) : \end{array} \begin{cases} x - 3y = 35 \\ x + 7y = 15 \end{cases} \qquad \begin{array}{l} (b_2) : \\ (p) : \end{array} \begin{cases} 3x + y = -15 \\ x + 7y = 15 \end{cases}$$

$C_1(29 ; -2)$ $\quad$ $C_2(-6 ; 3)$

④ Rayon des cercles

$$\delta(C_1, d_1) = \frac{|7 \cdot 29 + 2 - 5|}{\sqrt{50}} = \frac{200}{5\sqrt{2}} = \frac{40}{\sqrt{2}} \; ;$$

$(d_1) : 7x - y - 5 = 0$

$$\delta(C_2, d_2) = \frac{|7 \cdot (-6) - 3 - 5|}{\sqrt{50}} = \frac{50}{\sqrt{50}} = \sqrt{50} = 5\sqrt{2}$$

⑤ Cercles

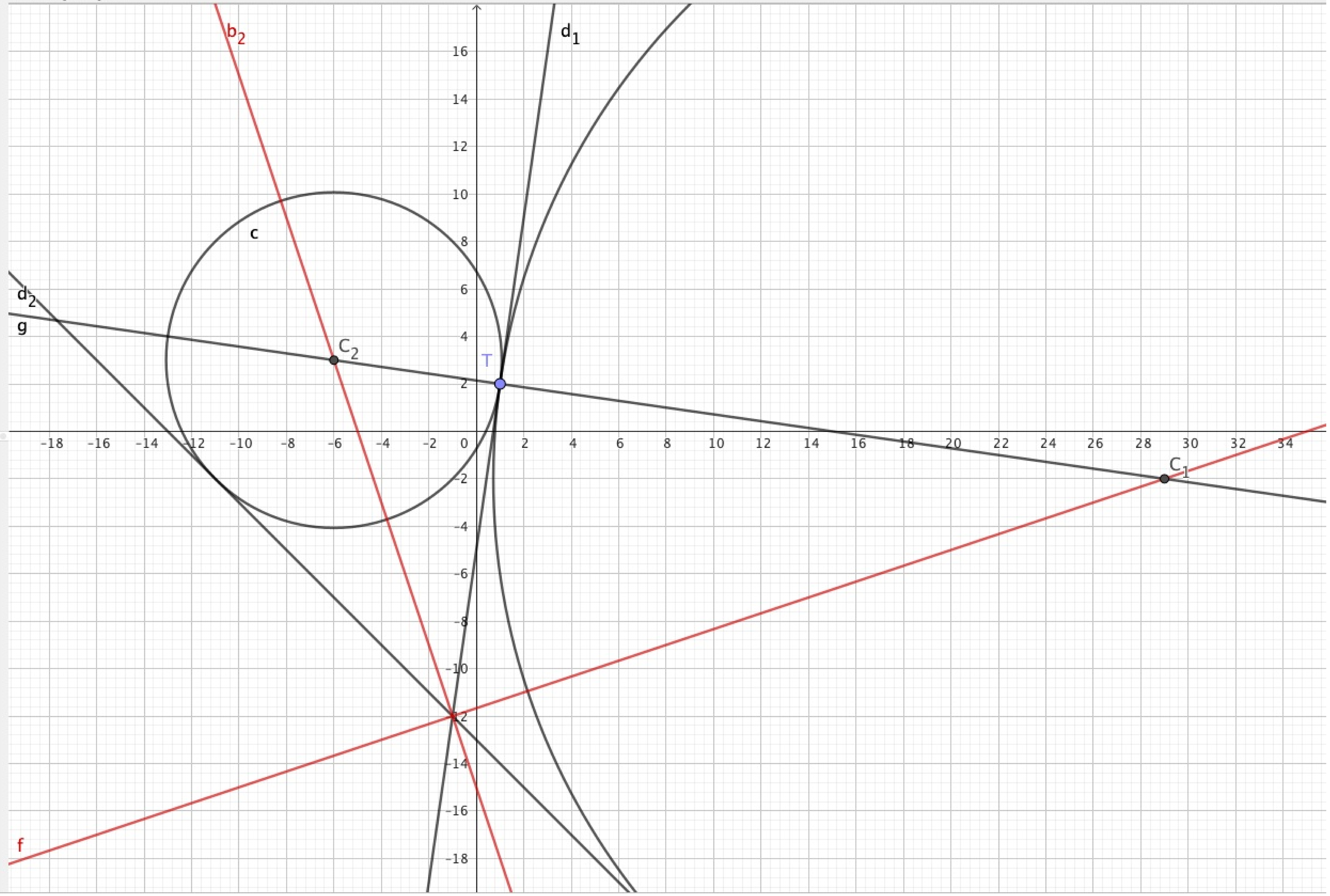
$(\gamma_1) : (x - 29)^2 + (y + 2)^2 = \left(\frac{40}{\sqrt{2}}\right)^2$ $\quad$ $(\gamma_2) : (x + 6)^2 + (y - 3)^2 = (5\sqrt{2})^2$

$(\gamma_1) : (x - 29)^2 + (y + 2)^2 = 800$ $\quad$ $(\gamma_2) : (x + 6)^2 + (y - 3)^2 = 50$

Algèbre

- d_1: $y = 7x - 5$
- d_2: $x + y = -13$
- $T = (1, 2)$
- b_2: $y = -3x - 15$
- f: $y = 0.33x - 11.67$
- g: $-x - 7y = -15$
- $C_2 = (-6, 3)$
- $C_1 = (29, -2)$
- c: $(x + 6)^2 + (y - 3)^2 = 50$
- d: $(x - 29)^2 + (y + 2)^2 = 800$

Graphique

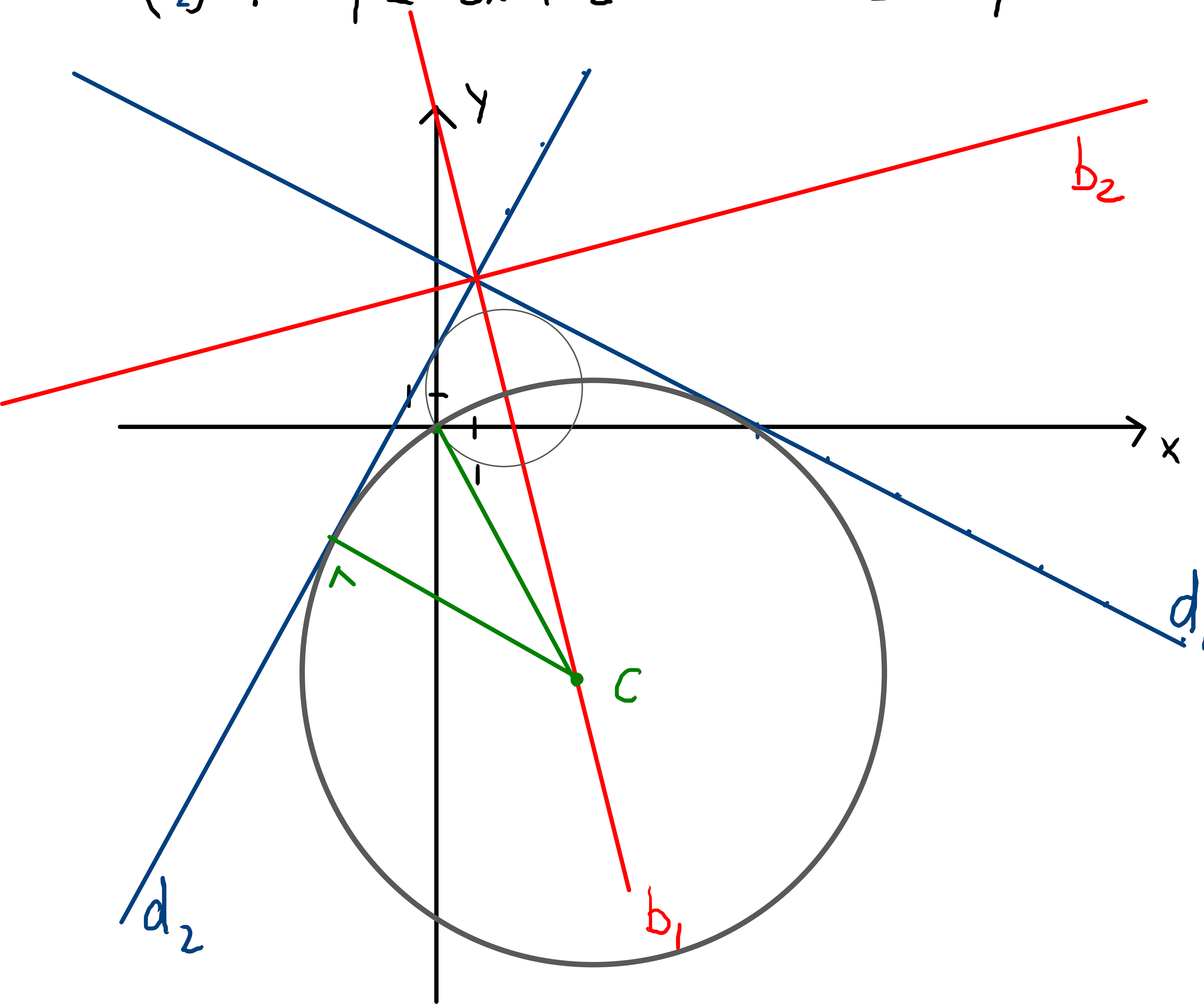


3.3.13 Déterminer les équations des cercles passant par l'origine et qui sont tangents aux droites $x + 2y = 9$ et $y = 2x + 2$.

(d_1) : $x + 2y = 9$ $\qquad x + 2y - 9 = 0$ $\qquad m_1 = -\frac{1}{2}$ $\qquad A_1(9; 0)$

(d_2) : $y = 2x + 2$ $\qquad 2x - y + 2 = 0$ $\qquad m_2 = 2$ $\qquad B_1(0; 2)$

$\left.\begin{matrix} m_1 = -\frac{1}{2} \\ m_2 = 2 \end{matrix}\right\} a \perp b$



La figure montre qu'on cherche deux cercles

① <u>bissectrice b_1 de pente négative</u>

$$\frac{x + 2y - 9}{\sqrt{5}} = \pm \frac{2x - y + 2}{\sqrt{5}}$$

"$-$": (b_1) : $3x + y - 7 = 0$

"$+$": (b_2) : $x - 3y + 11 = 0$

② Soit C le centre du cercle cherché

(i) $C \in b_1$

(ii) $\|\vec{OC}\| = R$ rayon du cercle

(iii) $\delta(C, d_1) = R$

(i) $C \in b_1$, posons $C(\alpha; \beta)$. Donc $3\alpha + \beta - 7 = 0 \Rightarrow \beta = -3\alpha + 7$

Donc $C(\alpha; -3\alpha + 7)$

(ii) $\vec{OC} = \begin{pmatrix} \alpha \\ -3\alpha+7 \end{pmatrix}$, $\|\vec{OC}\| = \sqrt{\alpha^2 + (-3\alpha+7)^2}$

$= \sqrt{10\alpha^2 - 42\alpha + 49} = R$

(iii) $\delta(C, d_1) = \dfrac{|\alpha - 6\alpha + 14 - 9|}{\sqrt{5}} = \dfrac{|-5\alpha + 5|}{\sqrt{5}} = R$

On résout l'équation :

$$\sqrt{10\alpha^2 - 42\alpha + 49} = \frac{|-5\alpha+5|}{\sqrt{5}} \qquad \Big|\ (\ )^2$$

$$10\alpha^2 - 42\alpha + 49 = \frac{25\alpha^2 - 50\alpha + 25}{5}$$

$$50\alpha^2 - 210\alpha + 245 = 25\alpha^2 - 50\alpha + 25$$

$$25\alpha^2 - 160\alpha + 220 = 0 \qquad \Big|\ \div 5$$

$$5\alpha^2 - 32\alpha + 44 = 0$$

$$(5\alpha - 22)(\alpha - 2) = 0$$

Si $\alpha = 2$, $\beta = -6 + 7 = 1 \Rightarrow C_1(2; 1)$

Si $\alpha = \frac{22}{5}$, $\beta = \frac{-66}{5} + 7 = \frac{-31}{5} \Rightarrow C_2\left(\frac{22}{5}; \frac{-31}{5}\right)$

Les rayons:

$r_1 = \dfrac{|-10 + 5|}{\sqrt{5}} = \sqrt{5}$; $r_2 = \dfrac{|-22 + 5|}{\sqrt{5}} = \dfrac{17}{\sqrt{5}}$

Finalement les cercles :

$$\boxed{\begin{aligned} (\gamma_1) &: (x-2)^2 + (y-1)^2 = 5 \\ (\gamma_2) &: \left(x - \tfrac{22}{5}\right)^2 + \left(y + \tfrac{31}{5}\right)^2 = \tfrac{289}{5} \end{aligned}}$$