21.02.24

$$(\sin(x))' = \cos(x)$$

$$(\cos(x))' = -\sin(x)$$

$$(\tan(x))' = 1 + \tan^2(x) = \frac{1}{\cos^2(x)}$$

2.9.12 Calculer la dérivée de chacune des fonctions suivantes :

a) $f(x) = \sin(x) + 2\cos(x)$

b) $f(x) = \tan(x) - x$

c) $f(x) = \dfrac{1}{\sin(x)}$

d) $f(x) = \dfrac{\sin(x)}{1+\cos(x)}$

c) $\left(\frac{1}{v}\right)' = \frac{-v'}{v^2}$; d) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

a) $f'(x) = \cos(x) - 2\sin(x)$

b) $f'(x) = 1 + \tan^2(x) - 1 = \tan^2(x)$

3ème

$$\boxed{\int \tan^2(x)\,dx = \tan(x) - x}$$

c) $f'(x) = \dfrac{-\cos(x)}{\sin^2(x)}$

d) $u = \sin(x)$; $u' = \cos(x)$

$v = 1 + \cos(x)$; $v' = -\sin(x)$

$$f'(x) = \frac{\cos(x)(1+\cos(x)) - \sin(x)\cdot(-\sin(x))}{(1+\cos(x))^2} = \frac{\cos(x) + \underbrace{\cos^2(x) + \sin^2(x)}_{=1}}{(1+\cos(x))^2}$$

$$= \frac{1+\cos(x)}{(1+\cos(x))^2} = \frac{1}{1+\cos(x)}$$

h) $f(x) = \sin^2(x) = \left(\sin(x)\right)^2$

$$f'(x) = 2\sin(x) \cdot \underbrace{\left(\sin(x)\right)'}_{\cos(x)} = 2\sin(x)\cos(x) \underset{\substack{\uparrow \\ \text{CRM}}}{=} \sin(2\alpha)$$

g) $f(x) = \sin(2x) = \cos(2x) \cdot \underbrace{(2x)'}_{\text{dérivée interne}} = 2\cos(2x)$

CRM

$$\left. \underset{\substack{\uparrow \\ \sin(y)}}{(g} \circ \underset{\substack{\nwarrow \\ \circ 2x}}{f})'(x) = g'(f(x)) \cdot f'(x) \right| = \left[g(f(x))\right]'$$

k) $f(x) = \tan^5(8x) = \left[\tan(8x)\right]^5$

$$x \longmapsto 8x \longmapsto \tan(8x) \longmapsto (\tan(8x))^5$$

$$f'(x) = 5\left(\tan(8x)\right)^4 \left(\tan(8x)\right)'$$

$$= 5\tan^4(8x)\left(1+\tan^2(8x)\right)(8x)'$$

$$= 40\tan^4(8x)\left(1+\tan^2(8x)\right)$$

1) $f(x) = \sqrt{1 - \tan(2x)}$

$$(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$$

$$f'(x) = \frac{(1-\tan(2x))'}{2\sqrt{1-\tan(2x)}} = \frac{-(1+\tan^2(2x))\cdot 2}{2\sqrt{1-\tan(2x)}} = \frac{-\cancel{2}(1+\tan^2(2x))}{\cancel{2}\sqrt{1-\tan(2x)}}$$

$$= \frac{-(1+\tan^2(2x))}{\sqrt{1-\tan(2x)}}$$

2.9.14 Déterminer les coefficients a, b et c de la fonction $f(x) = -x^3 + ax^2 + bx + c$, sachant que $f(2) = -1$, $f'(2) = 0$ et $f''(2) = 0$.

$f(2) = -1 \quad : \quad \underline{-8 + 4a + 2b + c = -1}$ (3)

$f'(2) = 0 \quad ; \quad f'(x) = -3x^2 + 2ax + b \quad ; \quad f'(2) = \underline{-12 + 4a + b = 0}$ (2)

$f''(2) = 0 \quad ; \quad (f'(x))' = f''(x) = -6x + 2a \quad ; \quad f''(2) = \underline{-12 + 2a = 0}$ (1)

(1) $\quad 2a = 12 \quad \Rightarrow \quad \underline{a = 6}$

(2) $\quad -12 + 24 + b = 0 \quad \Rightarrow \quad \underline{b = -12}$

(3) $\quad -8 + 24 - 24 + c = -1 \quad \Rightarrow \quad \underline{c = 7}$

2.9.15 Former l'équation de la tangente au graphe de f en son point d'abscisse a, si :

a) $f(x) = 1 + 2x - x^3$, $a = 1$

b) $f(x) = \dfrac{x+3}{x}$, $a = 3$

c) $f(x) = \sqrt{2x+1}$, $a = 4$

d) $f(x) = \dfrac{\sin(x)}{\sin(x) + \cos(x)}$, $a = 0$

a) la pente de la tangente en $a = 1$ est égale à $f'(1)$

$f'(x) = 2 - 3x^2$; $\boxed{f'(1) = -1} = m$

tangente : $y = -x + h$

Le point sur $y = f(x)$: $A(1 ; 2)$

Donc $2 = -1 + h \Rightarrow h = 3$

L'équation de la tangente : $y = -x + 3$