29.09.23

$a > 0$, $a \neq 1$

$$\boxed{a^x = y \qquad \log_a(y) = x}$$

4.2.2 Calculer à la main :

a) $\log_3(1) = 0$ b) $\log_2(8) = 3$ c) $\log_2(64) = 6$ d) $\log_2(1'024) = 10$

e) $\log_5(5) = 1$ f) $\log_3(\sqrt{3})$ g) $\log_{243}(1/243)$ h) $\log_3(27) = 3$

i) $\log(1'000) = 3$ j) $\log_4(\sqrt{2})$ k) $\log_{1/8}(64)$ l) $\log_5(0,04)$

m) $\log_3(\sqrt[4]{27})$ n) $\ln(e^2) = 2$ o) $\log_a(a) = 1$ p) $\log_a(a^3) = 3$

q) $\log(10000) = 4$ r) $\ln(e) = 1$ s) $\log_2(1/8)$ t) $\log_3(\sqrt[4]{3})$

u) $\log(200) - \log(2)$ v) $\log_6(4) + \log_6(9)$ w) $\log_5(1) = 0$ x) $\cancel{\log(-1)}$

y) $\log(0.0001) = -4$ z) $\cancel{\ln(0)}$

$$\log_{10}(x) = \log(x)$$

$$\log_e(x) = \ln(x)$$

f) $\log_3(\sqrt{3})$

g) $\log_{243}(1/243) = \log_{243}\left(243^{-1}\right) = -1$

j) $\log_4(\sqrt{2})$

k) $\log_{1/8}(64)$ $= \log_{\frac{1}{8}}\left(\left(\frac{1}{8}\right)^{-2}\right) = -2$

l) $\log_5(0,04)$ $\log_5\left(\frac{4}{100}\right) = \log_5\left(\frac{1}{25}\right) = -2$

m) $\log_3(\sqrt[4]{27}) = \frac{3}{4}$

s) $\log_2(1/8) = -3$

t) $\log_3(\sqrt[4]{3}) = \frac{1}{4}$

u) $\log(200) - \log(2)$

v) $\log_6(4) + \log_6(9)$

f) $\log_3(\sqrt{3}) = \log_3\left(3^{\frac{1}{2}}\right) = \frac{1}{2}$

j) $\log_4(\sqrt{2}) = \log_4\left(\sqrt{\sqrt{4}}\right) = \log_4\left(\left(4^{\frac{1}{2}}\right)^{\frac{1}{2}}\right) = \log_4\left(4^{\frac{1}{4}}\right) = \frac{1}{4}$

$$\boxed{\log_a\left(a^x\right) = x}$$

a) $\log_{10}(0,1) = 0,1 \iff 10^{-1} = 0,1$

b) $\log_2(32) = 5 \iff 2^{5} = 32$

c) $\log_3(243) = 5 \iff 3^{5} = 243$

d) $\log_{\frac{1}{5}}(125) = -3 \iff \left(\frac{1}{5}\right)^{-3} = 125$ $\qquad \left(\frac{1}{5}\right)^{-1}$

e) $\log_4(8) = \frac{3}{2} \iff 4^{\frac{3}{2}} = 8$

$\left(\sqrt{4}\right)^{3}$

Bases usuelles

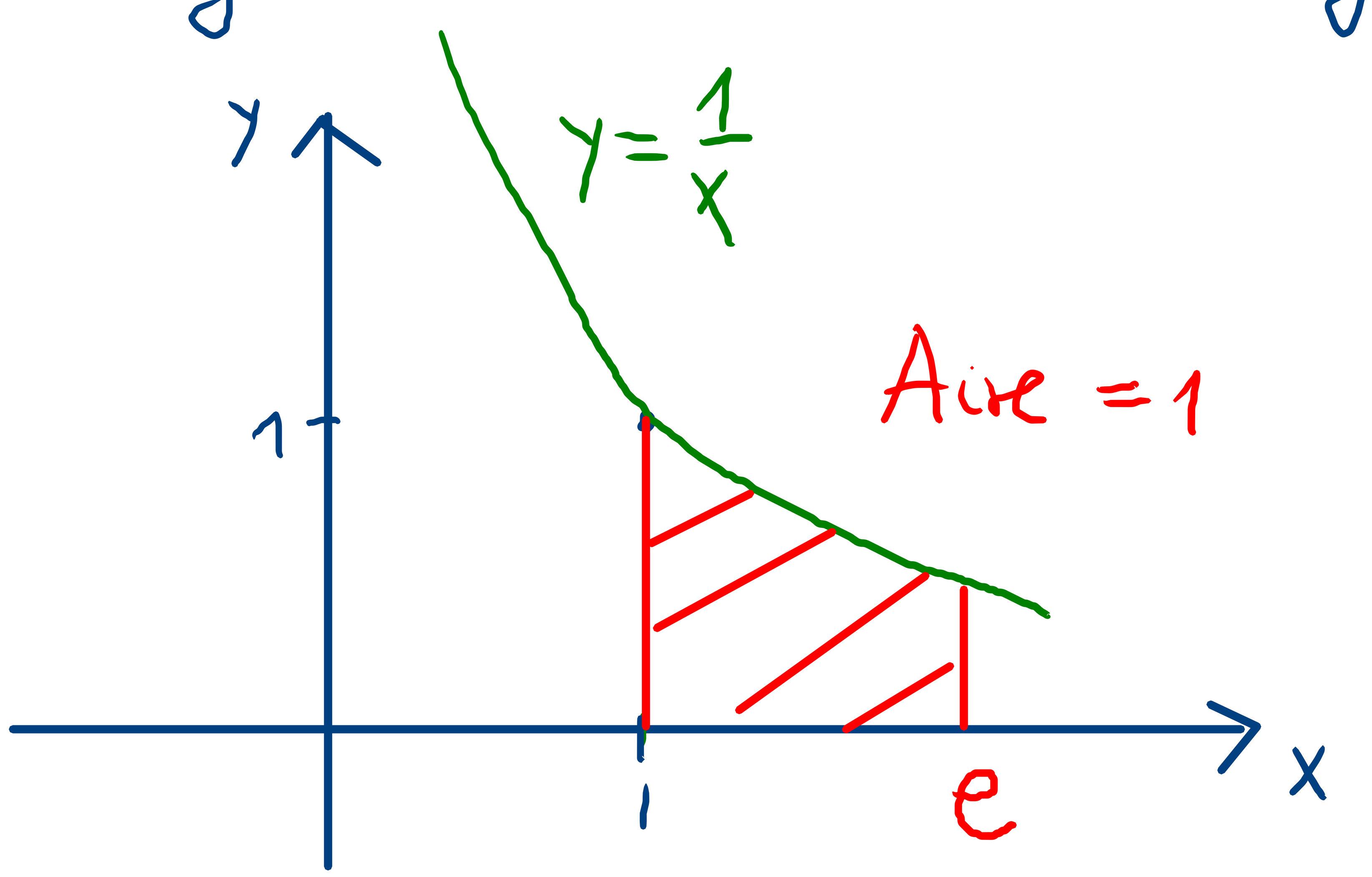
Nous utilisons deux bases :

- Base 10 : $\log_{10} \Rightarrow \log$

- Base $e \cong 2.718281828459045$

$$\log_e \Rightarrow \ln$$

log. naturel ou log népérien (John Napier)



$$\int_1^e \frac{1}{x}\,dx = e$$

Propriétés

Soit $a > 0$, $a \neq 1$.

1) $a^{\log_a(y)} = y$, $y > 0$

2) $\log_a(a^x) = x$, $x \in \mathbb{R}$

3) $\log_a(1) = 0$

4) $\log_a(a) = 1$

Soit u, v tels que $\log_a(x) = u$ et $\log_a(y) = v$, $x, y > 0$

On a $\log_a(x) = u \Leftrightarrow a^u = x$

$\log_a(y) = v \Leftrightarrow a^v = y$

5) $\log_a(xy) = \log_a(a^u \cdot a^v) = \log_a(a^{u+v}) \underset{2)}{=} u + v = \log_a(x) + \log_a(y)$

$\log_a(xy) = \log_a(x) + \log_a(y)$

6) $0 = \log_a(1) = \log_a\left(x \cdot \frac{1}{x}\right) \underset{5)}{=} \log_a(x) + \log_a\left(\frac{1}{x}\right)$

$\log_a\left(\frac{1}{x}\right) = -\log_a(x)$

7) $\log_a(x^r) = \log_a\left((a^u)^r\right) = \log_a(a^{u \cdot r}) = u \cdot r = \log_a(x) \cdot r$

$\log_a(x^r) = r \cdot \log_a(x)$

Propriétés

Soit $a > 0$, $a \neq 1$.

1) $a^{\log_a(y)} = y$, $y > 0$

2) $\log_a(a^x) = x$, $x \in \mathbb{R}$

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Soit u, v tels que $\log_a(x) = u$ et $\log_a(y) = v$, $x, y > 0$

On a $\log_a(x) = u \Leftrightarrow a^u = x$

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5) $\log_a(xy) = \log_a(a^u \cdot a^v) = \log_a(a^{u+v}) \underset{2)}{=} u + v = \log_a(x) + \log_a(y)$

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7) $\log_a(x^r) = \log_a((a^u)^r) = \log_a(a^{u \cdot r}) = u \cdot r = \log_a(x) \cdot r$

$$\log_a(x^r) = r \cdot \log_a(x)$$