1.1.13

e) $f(x) = \dfrac{e^x + 2}{1 - 3e^x}$

① ED(f) : condition : $1 - 3e^x \neq 0$

zéro(s) du dénominateur :

$3e^x = 1 \Leftrightarrow e^x = \frac{1}{3} \Leftrightarrow x = \ln\left(\frac{1}{3}\right) \Leftrightarrow x = \ln(1) - \ln(3)$

$\Leftrightarrow x = -\ln(3)$

$ED(f) = \mathbb{R} - \{-\ln(3)\}$

② Aucune parité, ED(f) pas symétrique par rapport à l'origine.

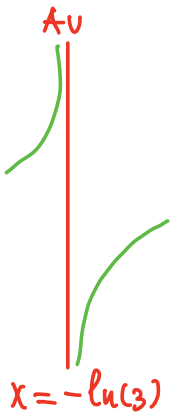
③ Signe de f :

| x | | $-\ln(3)$ | |
|---|---|---|---|
| $e^x + 2$ | + | ‖ | + |
| $1 - 3e^x$ | + | ‖ | − |
| $f(x)$ | + | ‖ | − |

$-\ln(3) \cong -1{,}10$

④ AV : $\lim\limits_{x \to -\ln(3)} \dfrac{e^x+2}{1-3e^x} = \text{“}\frac{2{,}3}{0}\text{”} \infty$ $\Rightarrow$ $x = -\ln(3)$ AV

$\Rightarrow$
$\lim\limits_{\substack{x \to -\ln(3) \\ <}} f(x) = +\infty$
$\lim\limits_{\substack{x \to -\ln(3) \\ >}} f(x) = -\infty$



AHD:

$$\lim_{x\to+\infty} \frac{e^x+2}{1-3e^x} = \lim_{x\to+\infty} \frac{e^x\left(1+\frac{2}{e^x}\right)}{e^x\left(-3+\frac{1}{e^x}\right)} = -\frac{1}{3} \Rightarrow \text{AHD}: y=-\frac{1}{3}$$

($\frac{2}{e^x} \to 0_f$, $\frac{1}{e^x} \to 0_f$)

AHG:

$$\lim_{x\to-\infty} \frac{e^x+2}{1-3e^x} = 2 \Rightarrow \text{AHG}: y=2$$

($e^x \to 0_f$, $3e^x \to 0_f$)

⑤ Croissance:

$u = e^x+2$ ; $u' = e^x$

$v = 1-3e^x$ ; $v' = -3e^x$

$$f'(x) = \frac{e^x(1-3e^x)-(e^x+2)\cdot(-3e^x)}{(1-3e^x)^2}$$

$$= \frac{e^x\left[1-3e^x+3e^x+6\right]}{(1-3e^x)^2} = \frac{7e^x}{(1-3e^x)^2}$$

| $x$ | $-\ln(3)$ | |
|---|---|---|
| $f'(x)$ | $+$ | $+$ |
| $f(x)$ | ↗ | ↗ |

La fonction est strictement croissante.

$$ED(f') = \mathbb{R} - \{-\ln(3)\}$$

## ⑥ Courbure

$u = 7e^x$ ; $u' = 7e^x$

$v = (1-3e^x)^2$ ; $v' = 2(1-3e^x)\cdot(-3e^x) = -6e^x(1-3e^x)$

$$f''(x) = \frac{7e^x(1-3e^x)^2 - 7e^x(-6e^x)(1-3e^x)}{(1-3e^x)^4}$$

$$= \frac{7e^x(1-3e^x)\left[1-3e^x+6e^x\right]}{(1-3e^x)^{4\,3}}$$

$$= \frac{7e^x(1+3e^x)}{(1-3e^x)^3}$$

$ED(f'') = \mathbb{R} - \{-\ln(3)\}$

| $x$ | | $-\ln(3)$ | |
|---|---|---|---|
| $f''(x)$ | + | ‖ | − |
| $f(x)$ | ∪ | ‖ | ∩ |

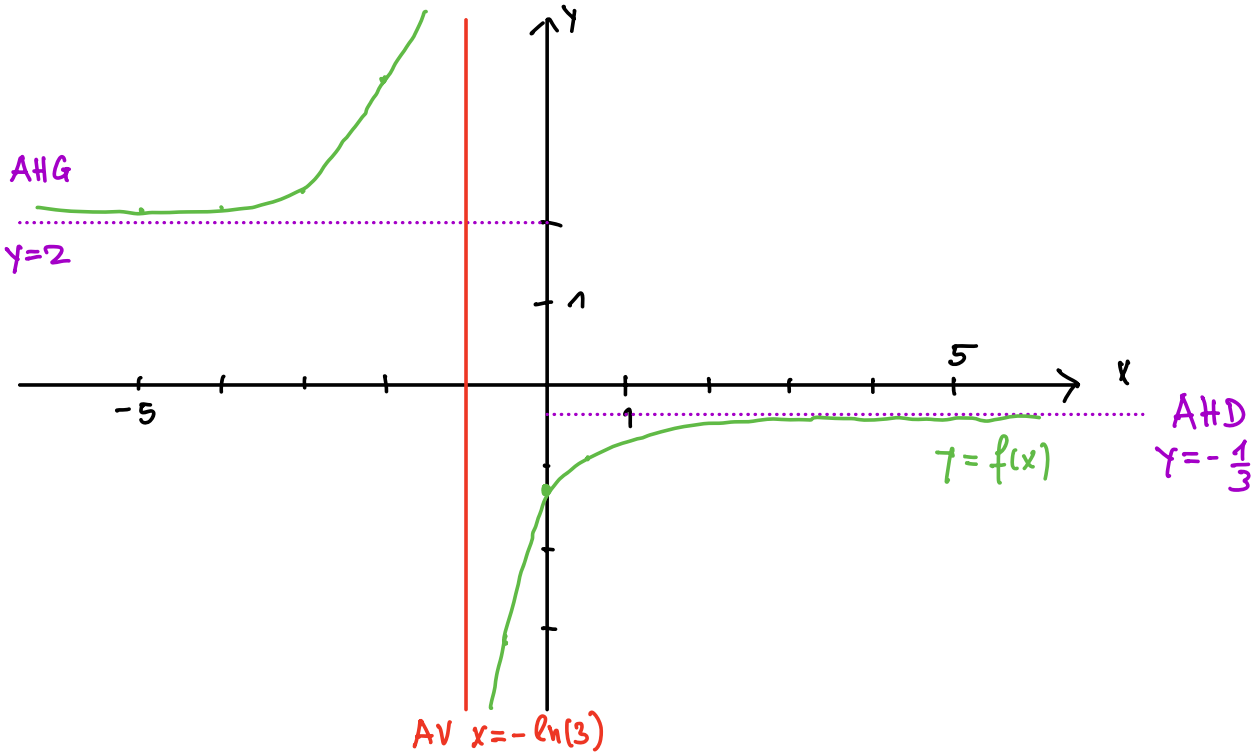
pas de point d'inflexion

## ⑧ Graphique

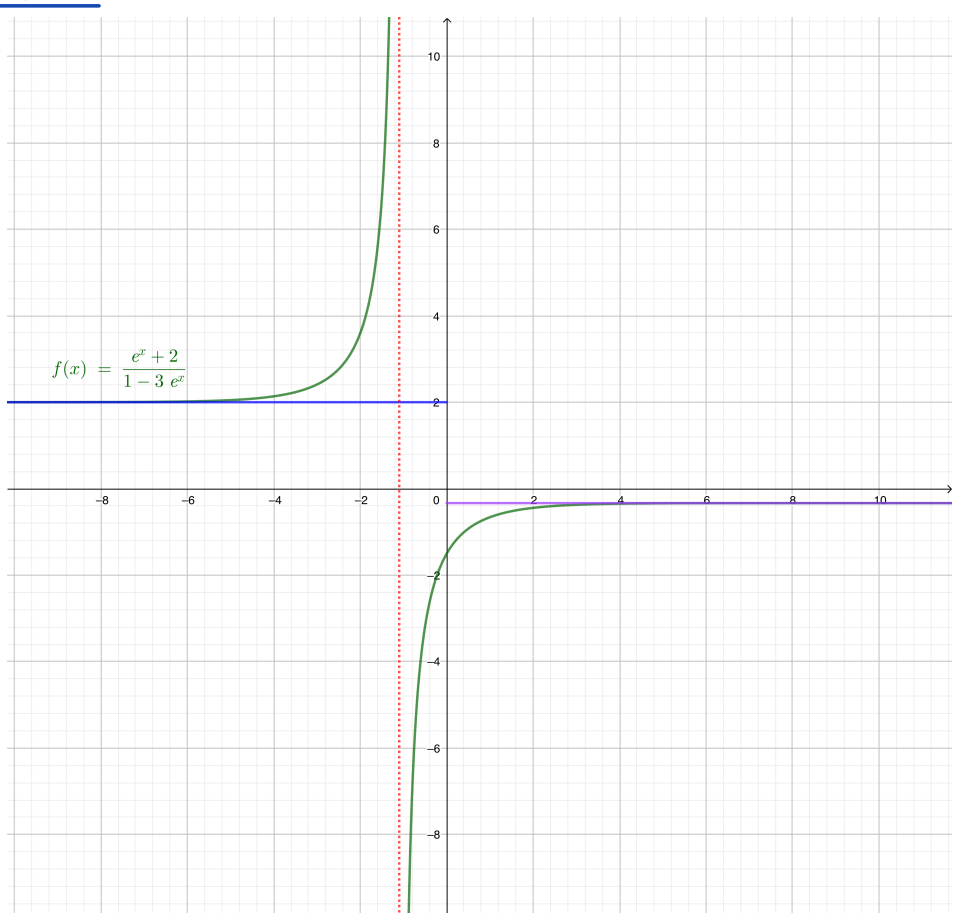
$f(0) = \frac{3}{-2} = -1.5$

$f(-5) \cong 2.1$ $f(-3) \cong 2,4$ $f(0.5) \cong -0.9$

$f(-2) \cong 3,6$ $f(-4) \cong 2.1$ $f(-0.5) \cong -3.2$


AHG
y=2
y
1
5
x
-5
1
AHD
y=-1/3
y=f(x)
AV x=-ln(3)

Geogebra


$f(x) = \frac{e^x + 2}{1 - 3\,e^x}$
10
8
6
4
2
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6
8
10