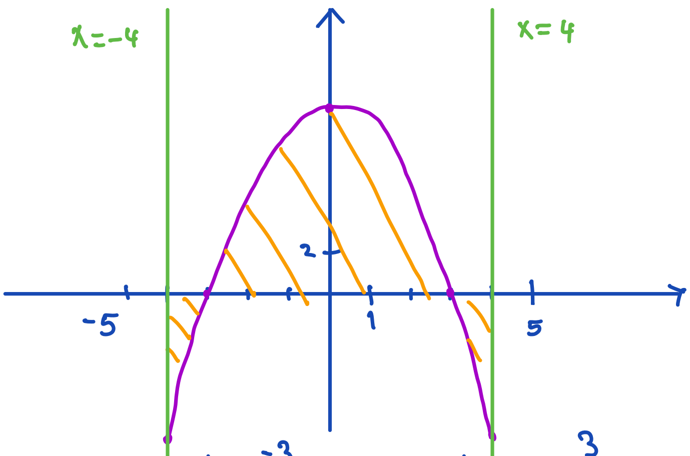
1.3.16 Calculer l'aire du domaine limité par la courbe d'équation $y = f(x)$, l'axe Ox et les droites $x = a$, et $x = b$:

a) $f(x) = 9 - x^2, \quad a = -4, b = 4$;

$$f(x) = 9 - x^2 = (3-x)(3+x) \qquad ED(f) = \mathbb{R}$$

x		-3		3	
$f(x)$	$-$	0	$+$	0	$-$

$$f(-4) = 9 - 16 = -7 \quad \text{et} \quad f(4) = -7$$



$$\int (9-x^2)\,dx = 9x - \frac{x^3}{3} + c = \frac{27x - x^3}{3} + c$$

Aire : $\left| \int_{-4}^{-3} (9-x^2)\,dx \right| + \int_{-3}^{3} (9-x^2)\,dx + \left| \int_{3}^{4} f(x)\,dx \right|$

$$= \left| \left[\frac{27x - x^3}{3} \right]_{-4}^{-3} \right| + \left[\frac{27x - x^3}{3} \right]_{-3}^{3} + \left| \left[\frac{27x - x^3}{3} \right]_{3}^{4} \right|$$

$$= \left| \frac{(-81+27) - (-108+64)}{3} \right| + \frac{(81-27) - (-81+27)}{3} + \left| \frac{(108-64) - (81-27)}{3} \right|$$

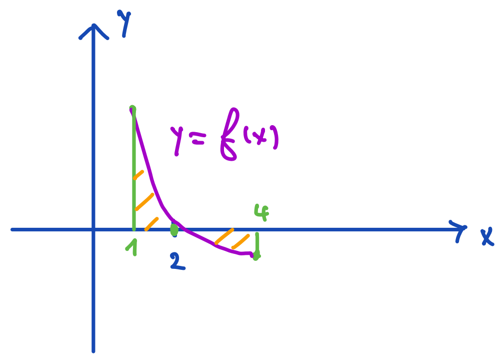
$$= \left| -\frac{10}{3} \right| + \frac{108}{3} + \left| -\frac{10}{3} \right| = \underline{\underline{\frac{128}{3}}}$$

b) $f(x) = \dfrac{4}{x^2} - 1, \quad a = 1,\ b = 4;$

$$f(x) = \frac{4}{x^2} - 1 = \frac{4 - x^2}{x^2} = \frac{(2-x)(2+x)}{x^2}$$

$ED(f) = \mathbb{R}^*$

x		-2		0		2	
$f(x)$	$-$	0	$+$	$\|$	$+$	0	$-$



$f(1) = 3 \quad , \quad f(4) = -\frac{3}{4}$

$$\int \left(\frac{4}{x^2} - 1\right) dx = \int \left(4x^{-2} - 1\right) dx = -4x^{-1} - x + c = \frac{-4}{x} - x + c$$

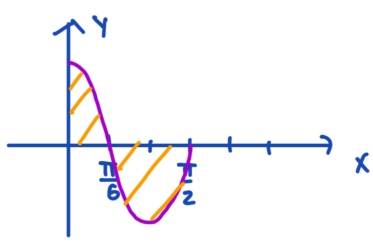
Aire : $\displaystyle\int_1^2 f(x)\,dx + \left| \int_2^4 f(x)\,dx \right| = \left[\frac{-4}{x} - x\right]_1^2 + \left| \left[\frac{-4}{x} - x\right]_2^4 \right|$

$$= (-2-2) - (-4-1) + \left| (-1-4) - (-2-2) \right| = 1 + 1 = \underline{\underline{2}}$$

c) $f(x) = \cos(3x), \quad a = 0,\ b = \dfrac{\pi}{2};$

$$\cos(3x) = 0 \iff \begin{cases} 3x = \frac{\pi}{2} + 2k\pi \\ 3x = \frac{3\pi}{2} + 2k\pi \end{cases} \iff \begin{cases} x = \frac{\pi}{6} + k \cdot \frac{2\pi}{3} \\ x = \frac{\pi}{2} + k \cdot \frac{2\pi}{3} \end{cases}$$

zéros sur $\left[0 ; \frac{\pi}{2}\right]$: $\frac{\pi}{6}$ et $\frac{\pi}{2}$



$$\int \cos(3x)\,dx = \frac{1}{3}\sin(3x) + c$$

Aire : $$\int_0^{\pi/6} \cos(3x)\,dx + \left| \int_{\pi/6}^{\pi/2} \cos(3x)\,dx \right| = \left[\frac{1}{3}\sin(3x)\right]_0^{\pi/6} + \left| \left[\frac{1}{3}\sin(3x)\right]_{\pi/6}^{\pi/2} \right|$$

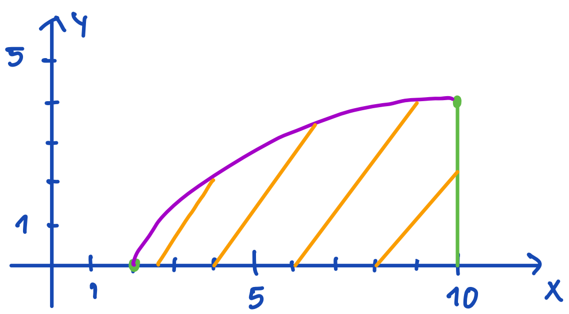
$$= \frac{1}{3}\left(\sin\left(\frac{\pi}{2}\right) - \sin(0)\right) + \left| \frac{1}{3}\left(\sin\left(\frac{3\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)\right) \right|$$

$$= \frac{1}{3}(1 - 0) + \left| \frac{1}{3}(-1-1) \right| = \frac{1}{3} + \frac{2}{3} = \underline{\underline{1}}$$

d) $f(x) = \sqrt{2x-4}, \quad a = 2,\ b = 10.$

$$2x - 4 \geq 0 \iff 2x \geq 4 \iff x \geq 2$$

$f(2) = 0$ et $f(10) = 4$



$f(x) \geq 0$ pour $x \geq 2$

$$\int \sqrt{2x-4}\,dx = \int (2x-4)^{1/2}\,dx = \frac{1}{3}(2x-4)^{3/2} + c = \frac{1}{3}\left(\sqrt{2x-4}\right)^3 + c$$

candidat : $K(2x-4)^{3/2}$

(candidat)' : $\frac{3}{2}K(2x-4)^{1/2} \cdot 2 = 3K(2x-4)^{1/2} \Rightarrow K = \frac{1}{3}$

$$\int_2^{10} \sqrt{2x-4}\,dx = \frac{1}{3}\left[\left(\sqrt{2x-4}\right)^3\right]_2^{10} = \frac{1}{3}\left(4^3 - 0\right) = \underline{\underline{\frac{64}{3}}}$$