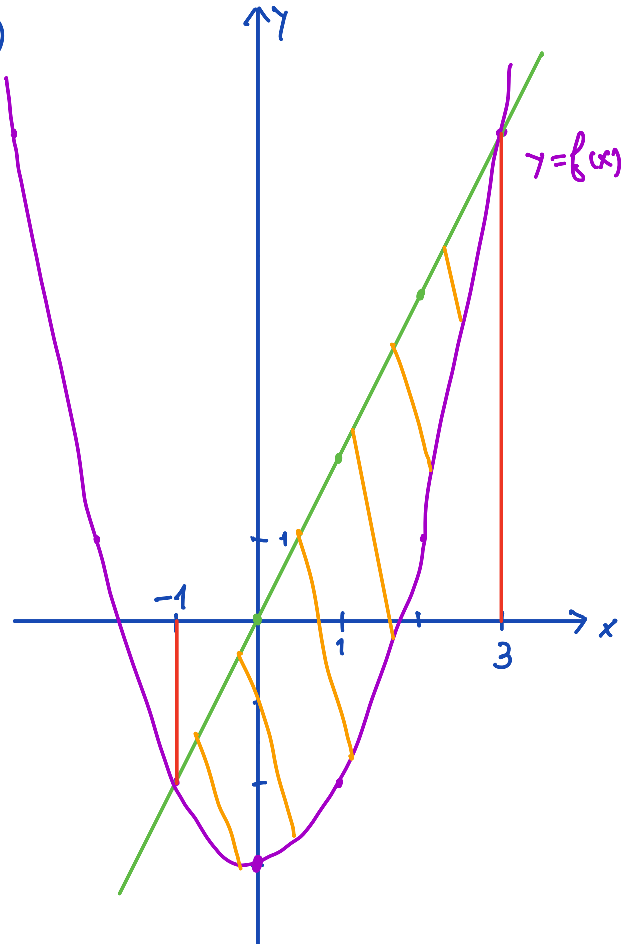
1.3.19 Calculer l'aire du domaine borné limité par les graphes des fonctions f et g :

a) $f(x) = x^2 - 3, \quad g(x) = 2x$

b) $f(x) = x^2, \quad g(x) = 8 - x^2$

a)



Intersection des deux courbes:

$$x^2 - 3 = 2x$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3 \qquad x = -1$$

Aire : $\left| \int_{-1}^{3} (x^2 - 3 - 2x)\, dx \right| = \left| \left[\frac{1}{3}x^3 - 3x - x^2 \right]_{-1}^{3} \right|$

$$= \left| \left(\frac{1}{3} \cdot 27 - 9 - 9\right) - \left(\frac{1}{3} \cdot (-1) + 3 - 1\right) \right| = \left| 9 - 9 - 9 + \frac{1}{3} - 2 \right|$$

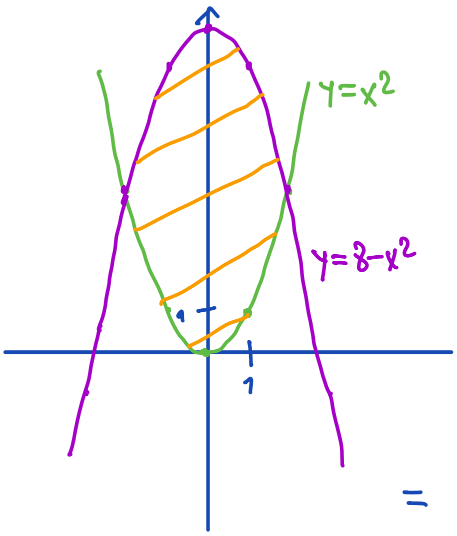
$$= \left| \frac{-27 + 1 - 6}{3} \right| = \frac{32}{3}$$

2) Intersection des deux courbes :

$$x^2 = 8 - x^2$$

$$2x^2 = 8 \iff x^2 = 4 \iff x^2 - 4 = 0 \iff (x-2)(x+2) = 0$$

$x = 2 \qquad x = -2$



Aire: $\left| \int_{-2}^{2} (8 - x^2 - x^2)\,dx \right| = \left| \int_{-2}^{2} (8 - 2x^2)\,dx \right|$

$$= \left[8x - \frac{2}{3}x^3 \right]_{-2}^{2} = \left(16 - \frac{16}{3}\right) - \left(-16 + \frac{16}{3}\right)$$

$$= 32 - \frac{32}{3} = \frac{96 - 32}{3} = \frac{64}{3}$$

c) $f(x) = x^3 - 5x^2 + 6x, \quad g(x) = x^3 - 7x^2 + 12x$

Intersection: $x^3 - 5x^2 + 6x = x^3 - 7x^2 + 12x$

$$2x^2 - 6x = 0$$

$$2x(x-3) = 0$$

$x = 0 \qquad x = 3$

Aire: $\left| \int_0^3 (x^3 - 5x^2 + 6x - x^3 + 7x^2 - 12x)\,dx \right|$

$$= \left| \int_0^3 (2x^2 - 6x)\,dx \right| = \left| \left[\frac{2}{3}x^3 - 3x^2 \right]_0^3 \right| = |18 - 27| = 9$$

d) $f(x) = x(6-2x^2), \quad g(x) = x(2-x^2)$

Intersection des deux courbes :

$$x(6-2x^2) = x(2-x^2)$$

$$x(6-2x^2) - x(2-x^2) = 0$$

$$x(6-2x^2-2+x^2) = 0$$

$$x(-x^2+4) = 0$$

$$x(x^2-4) = 0$$

$$x(x-2)(x+2) = 0$$

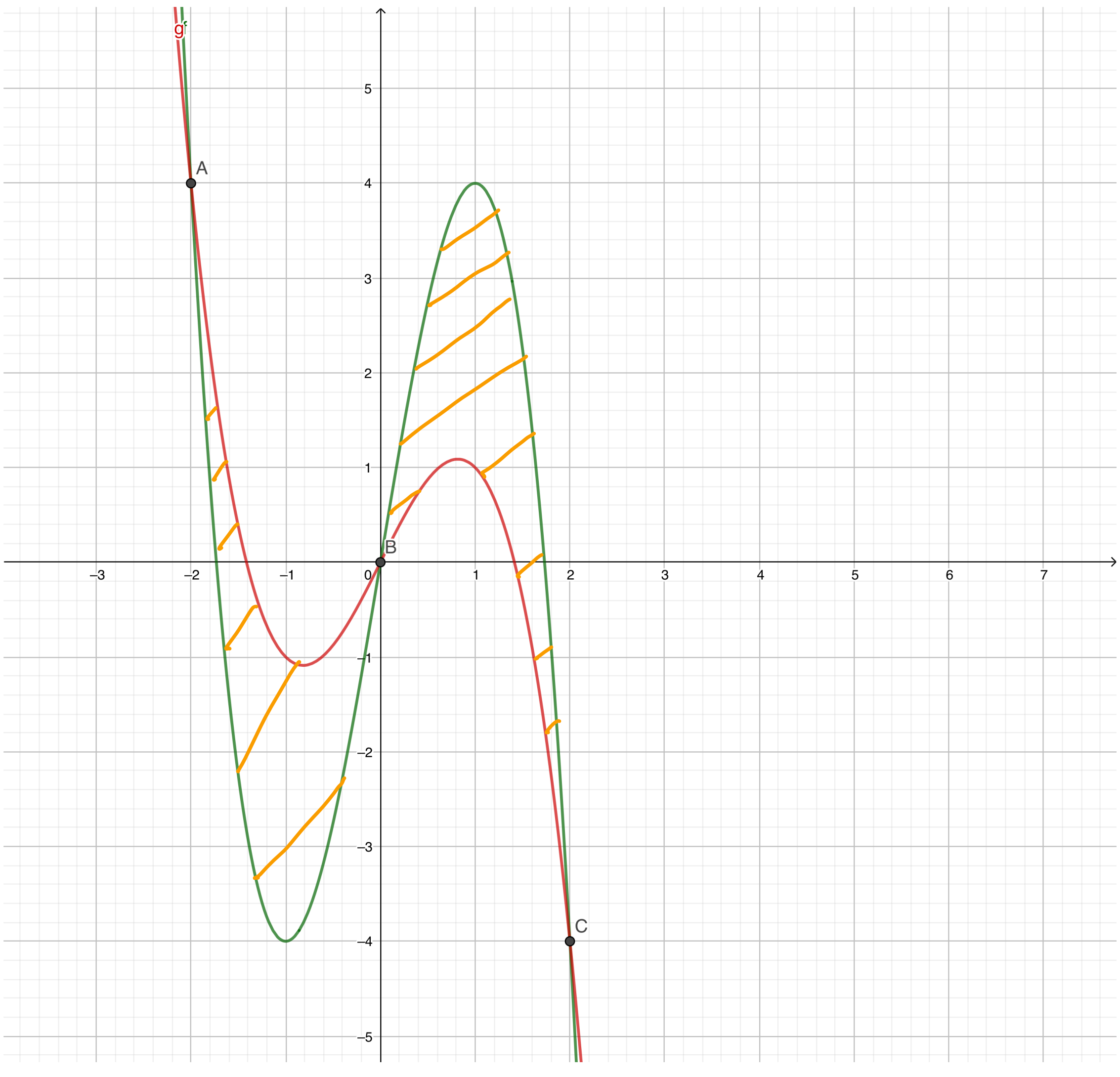
$$0 \qquad x=2 \qquad x=-2$$

Aire : $\left|\int_{-2}^{0} (6x-2x^3)-(2x-x^3)\,dx\right| + \left|\int_{0}^{2} (6x-2x^3)-(2x-x^3)\,dx\right|$

$$= \left|\int_{-2}^{0} (-x^3+4x)\,dx\right| + \left|\int_{0}^{2} (-x^3+4x)\,dx\right|$$

$$= \left|\left[-\frac{x^4}{4}+2x^2\right]_{-2}^{0}\right| + \left|\left[-\frac{x^4}{4}+2x^2\right]_{0}^{2}\right|$$

$$= \left|0-(-4+8)\right| + \left|-4+8-0\right| = 8$$


g
f
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