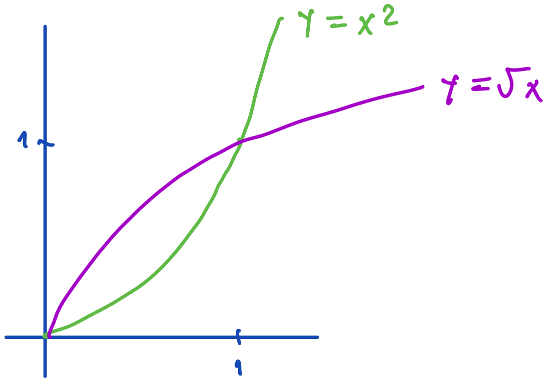
1.3.28 Le domaine délimité par les courbes d'équations $y = f(x)$, $y = g(x)$ et l'axe Ox tourne autour de cet axe. Calculer son volume :

a) $f(x) = \sqrt{x}$ et $g(x) = x^2$

b) $f(x) = x^2 - 2x + 6$ et $g(x) = -x^2 + 10$



Intersections:

$x^2 = \sqrt{x} \quad \Big| \; (\,)^2, \; x \geq 0$

$x^4 = x$

$x(x^3 - 1) = 0$

$x = 0 \qquad x = 1$

$$V_1 = \pi \int_0^1 (x^2)^2 \, dx = \pi \int_0^1 x^4 \, dx = \pi \frac{1}{5} x^5 \Big|_0^1 = \frac{\pi}{5}$$

$$V_2 = \pi \int_0^1 (\sqrt{x})^2 \, dx = \pi \int_0^1 x \, dx = \pi \frac{1}{2} x^2 \Big|_0^1 = \frac{\pi}{2}$$

$$V = \frac{\pi}{2} - \frac{\pi}{5} = \frac{5\pi - 2\pi}{10} = \underline{\frac{3\pi}{10}}$$

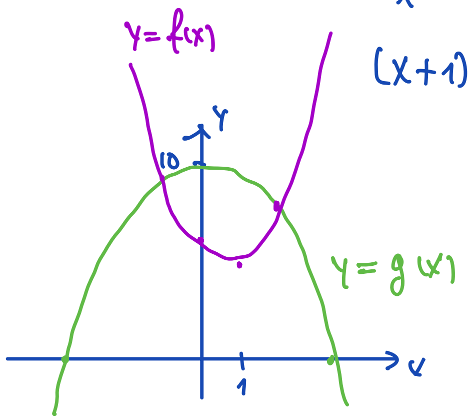
b) Intersections :

$x^2 - 2x + 6 = -x^2 + 10$

$2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$



$$V_1 = \pi \int_{-1}^{2} \left(-x^2+10\right)^2 dx = \pi \int_{-1} (x^4 - 20x^2 + 100)\, dx$$

$$V_2 = \pi \int_{-1}^{2} (x^2 - 2x + 6)^2 = \pi \int_{-1}^{2} (x^4 - 4x^3 + 16x^2 - 24x + 36)\, dx$$

$$V_1 - V_2 = \pi \int_{-1}^{2} (4x^3 - 36x^2 + 24x + 64)\, dx$$

$$= \pi \left[x^4 - 12x^3 + 12x^2 + 64x \right]_{-1}^{2}$$

$$= \pi \left[(16 - 96 + 48 + 128) - (1 + 12 + 12 - 64) \right]$$

$$= \pi\,(96 + 39) = \underline{135\pi}$$