1.3.6 Calculer :

a) $\int (3x^2 - 2x + 3)\, dx \quad = \quad \underline{x^3 - x^2 + 3x + C}$

b) $\int \frac{3x^4 - 3x^2 - 7}{4x^2}\, dx = \int \frac{3x^4}{4x^2} dx - \int \frac{3x^2}{4x^2} dx - \int \frac{7}{4x^2} dx$

$= \int \frac{3}{4} x^2 dx - \int \frac{3}{4} dx - \int \frac{7}{4} \frac{1}{x^2} dx$

$= \frac{3}{4} \int x^2 dx - \frac{3}{4} \int 1 \cdot dx - \frac{7}{4} \int x^{-2} dx$

$= \frac{3}{4} \cdot \frac{1}{3} x^3 - \frac{3}{4} x - \frac{7}{4} \frac{1}{\underbrace{-2+1}_{=-1}} x^{-1} + c$

$= \underline{\frac{1}{4} x^3 - \frac{3}{4} x + \frac{7}{4} \cdot \frac{1}{x} + c}$

c) $\int 7\sqrt[4]{x^3}\, dx \quad = 7 \int x^{3/4} dx = 7 \frac{1}{\frac{3}{4}+1} \cdot x^{3/4+1} + c$

$= 7 \cdot \frac{1}{\frac{7}{4}} \cdot x^{7/4} + c = \cancel{7} \cdot \frac{4}{\cancel{7}} \cdot \sqrt[4]{x^7} + c = \underline{4\sqrt[4]{x^7} + c}$

$= 4 \cdot \sqrt[4]{x^4 \cdot x^3} + c = \underline{4x\sqrt[4]{x^3} + c}$

d) $\int (\sqrt{x} - \sqrt[3]{x})\, dx \quad = \int x^{1/2} dx - \int x^{1/3} dx = \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} - \frac{1}{\frac{1}{3}+1} x^{\frac{1}{3}+1} + C$

$= \frac{1}{\frac{3}{2}} x^{3/2} - \frac{1}{\frac{4}{3}} x^{4/3} + c = \underline{\frac{2}{3} \sqrt{x^3} - \frac{3}{4} \sqrt[3]{x^4} + c}$

e) $\int (2\sin(x) - 3\cos(x))\, dx \quad = 2 \int \sin(x)\, dx - 3 \int \cos(x)\, dx$

$= -2\cos(x) - 3\sin(x) + c$

f) $\int \cos(2x)\,dx = \frac{1}{2}\sin(2x) + c$

candidat : $K \sin(2x)$

(candidat)' : $K\cdot\cos(2x)\cdot(2x)' = 2K\cos(2x) \Rightarrow 2K = 1 \Leftrightarrow K = \frac{1}{2}$

g) $\int \left(\frac{5}{\cos^2(x)} + 5\cos(x)\right) dx = 5\boxed{\int \frac{1}{\cos^2(x)}\,dx} + 5\int \cos(x)\,dx$

$\Downarrow$ Formulaire

$= 5\cdot \tan(x) + 5\sin(x) + c$

h) $\int \left(8\sin(x) + \frac{4}{\sqrt{2x}}\right) dx = 8\int \sin(x)\,dx + 4\int (2x)^{-1/2}\,dx$

$\int (2x)^{-1/2}\,dx = (2x)^{\frac{1}{2}}$

candidat : $K\cdot(2x)^{\frac{1}{2}}$

(candidat)' : $K\cdot\frac{1}{2}(2x)^{-\frac{1}{2}}\cdot(2x)' = K\cdot\frac{1}{2}(2x)^{\frac{1}{2}}\cdot 2 = K\cdot(2x)^{-\frac{1}{2}} \Rightarrow K=1$

$= -8\cos(x) + 4\cdot(2x)^{\frac{1}{2}} + c$

$= -8\cos(x) + 4\sqrt{2x} + c$

i) $\int (3x^2-7)^2\,dx = \int (9x^4 - 42x^2 + 49)\,dx$

$= \frac{9}{5}x^5 - \frac{42}{3}x^3 + 49x + c = \underline{\frac{9}{5}x^5 - 14x^3 + 49x + c}$

j) $\int \sqrt{x}(x^2-5)\,dx = \int (x^{\frac{1}{2}}\cdot x^2 - 5x^{\frac{1}{2}})\,dx = \int x^{5/2}\,dx - 5\int x^{1/2}\,dx$

$= \frac{1}{\frac{5}{2}+1}\,x^{\frac{5}{2}+1} - 5\,\frac{1}{\frac{1}{2}+1}\,x^{\frac{1}{2}+1} + c$

$= \frac{1}{\frac{7}{2}}\,x^{7/2} - 5\,\frac{1}{\frac{3}{2}}\,x^{\frac{3}{2}} + c$

$$= \frac{2}{7}\sqrt{x^7} - \frac{10}{3}\sqrt{x^3} + c$$

k) $\int (3x-5)^6\,dx \quad = \frac{1}{21}(3x-5)^7 + c$

candidat : $K(3x-5)^7$

(candidat)' : $K \cdot 7(3x-5)^6 \cdot \underbrace{(3x-5)'}_{3} = 21K(3x-5)^6$

$$\Rightarrow 21K = 1 \iff K = \frac{1}{21}$$

l) $\int \frac{12}{(4-3x)^4}\,dx \quad = 12\int (4-3x)^{-4}\,dx$

candidat : $K(4-3x)^{-3}$

(candidat)' : $K \cdot (-3)(4-3x)^{-4} \cdot \underbrace{(4-3x)'}_{-3} = 9K(4-3x)^{-4}$

$$\Rightarrow 9K = 12 \Rightarrow K = \frac{12}{9} = \frac{4}{3}$$

$$= \frac{4}{3}(4-3x)^{-3} + c = \frac{4}{3}\,\frac{1}{(4-3x)^3} + c$$

m) $\int \sqrt[3]{(3x-8)^2}\,dx \quad = \int (3x-8)^{2/3}\,dx$

candidat : $K(3x-8)^{5/3}$

(candidat)' : $K \cdot \frac{5}{3}(3x-8)^{2/3} \cdot 3 = 5K(3x-8)^{2/3}$

$$\Rightarrow 5K = 1 \Rightarrow K = \frac{1}{5}$$

$$= \frac{1}{5}(3x-8)^{\frac{5}{3}} + c = \frac{1}{5}\sqrt[3]{(3x-8)^5} + c$$

n) $\int \frac{6}{\cos^2(3x)}\,dx = 6\int \frac{1}{\cos^2(3x)}\,dx = 2\tan(3x) + c$

candidat : $K \tan(3x)$

$(\text{candidat})'$: $K \cdot \frac{1}{\cos^2(3x)} \cdot 3 = 3K \cdot \frac{1}{\cos^2(3x)} \Rightarrow 3K = 6 \Rightarrow K = 2$

o) $\int x\sqrt{x^2+1}\,dx = \int x\,(x^2+1)^{\frac{1}{2}}\,dx = \frac{1}{3}(x^2+1)^{3/2} + C = \frac{1}{3}\sqrt{(x^2+1)^3} + c$

candidat : $K\,(x^2+1)^{\frac{3}{2}}$

$(\text{candidat})'$: $K \cdot \frac{3}{2}(x^2+1)^{\frac{1}{2}} \cdot 2x = 3K\,x\,(x^2+1)^{\frac{1}{2}}$

$\Rightarrow 3K = 1 \Rightarrow K = \frac{1}{3}$

p) $\int \frac{2x-1}{\sqrt{x^2-x-1}}\,dx = \int (2x-1)(x^2-x-1)^{-\frac{1}{2}}\,dx$

candidat : $K(x^2-x-1)^{\frac{1}{2}}$

$(\text{Candidat})'$: $K \cdot \frac{1}{2}(x^2-x-1)^{-\frac{1}{2}} \cdot (2x-1) = \frac{1}{2}K\,(2x-1)(x^2-x-1)$

$\Rightarrow \frac{1}{2}K = 1 \Rightarrow K = 2$

$= 2(x^2-x-1)^{\frac{1}{2}} + C = 2\sqrt{x^2-x-1} + c$