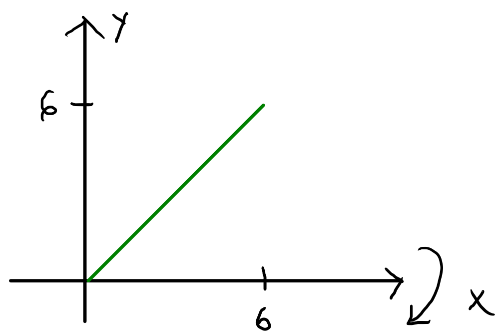
Solide de révolution

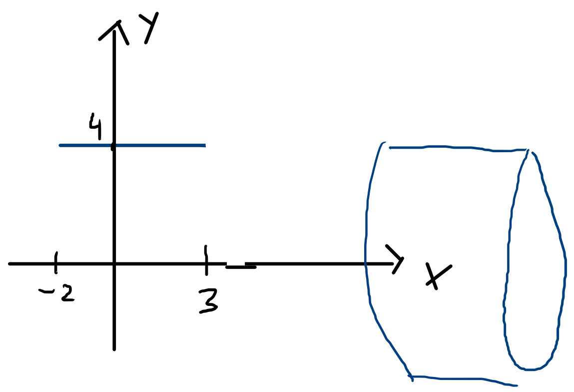
1) 

$f(x) = x \qquad 0 \leq x \leq 6$

On fait tourner autour de l'axe des x.

On obtient un cône de révolution

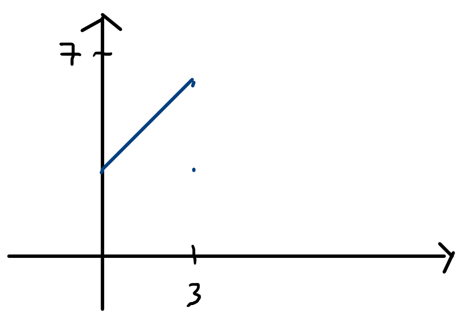
$V = \frac{1}{3} B \times h = \frac{1}{3} \cdot \pi 6^2 \cdot 6 = 72 \; [u^3]$

2) 

$f(x) = 4 \; , \quad -2 \leq x \leq 3$

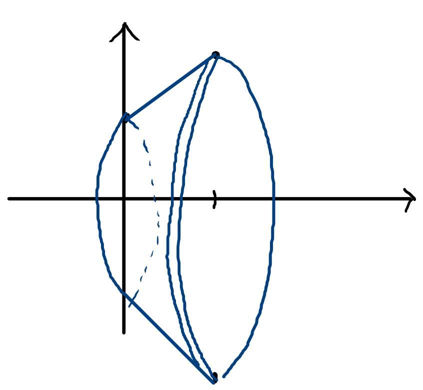
Le volume est un cylindre de révolution

$V = B \times h = \pi \cdot 4^2 \cdot 5 = 80\pi \quad [u^3]$

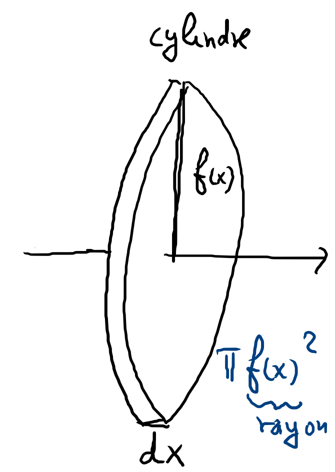
3) 

$f(x) = x + 3 \quad , \quad 0 \leq x \leq 4$

Le volume est un tronc de cône



$$\underbrace{\int_0^3 \pi (x+3)^2 \, dx}_{\text{cylindre infinitésimal}} = \pi \int_0^3 (x+3)^2 \, dx$$



Calculons $\int (x+3)^2 dx = \frac{1}{3}(x+3)^3 + c$

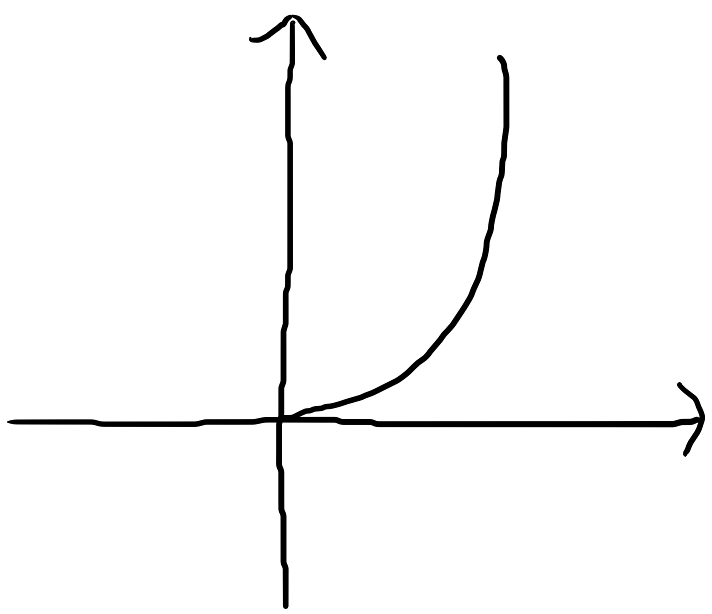
$$V = \frac{1}{3}(x+3)^3 \Big|_0^3 = \frac{1}{3} \cdot 6^3 - \frac{1}{3} 3^3 = 72 - 9 = 63 \quad [u^3]$$

Formule

$f(x) \geq 0$ pour $x \in [a, b]$

$$V = \pi \int_a^b f(x)^2 \, dx$$

4) $f(x) = x^2 \qquad 0 \le x \le 3$



$$V = \pi \int_0^3 (x^2)^2 \, dx = \pi \int_0^3 x^4 \, dx = \pi \left[\frac{1}{5} x^5 \right]_0^3$$

$$= \pi \, \frac{1}{5} \cdot 3^5 = \frac{729}{5} \pi$$

1.3.26

1.3.26 Le domaine délimité par la courbe d'équation $y = f(x)$, l'axe Ox et les droites $x = a$ et $x = b$ tourne autour de l'axe Ox. Esquisser le corps ainsi obtenu et calculer son volume :

a) $f(x) = x + 1,\quad a = 1, b = 3$

c) $f(x) = \dfrac{1}{x+1},\quad a = 1, b = 2$

a) $$V = \pi \int_1^3 (x+1)^2\, dx = \pi \left[\frac{1}{3}(x+1)^3\right]_1^3$$

$$= \pi\left(\frac{1}{3}\cdot 4^3 - \frac{1}{3}\cdot 2^3\right) = \pi\left(\frac{63}{3} - \frac{8}{3}\right) = \pi\,\frac{55}{3} = \frac{55}{3}\pi \quad [u^3]$$

b)

b) $f(x) = x^2,\quad a = 0, b = 4$

$$V = \pi \int_0^4 (x^2)^2\, dx = \pi \int_0^4 x^4\, dx = \pi\,\frac{1}{5}\left[x^5\right]_0^4 = \frac{\pi}{5}\left(4^5 - 0^5\right) = \frac{\pi}{5}\cdot 1024$$

$$= \frac{1024}{5}\pi$$

c) $$V = \pi \int_1^2 \left(\frac{1}{x+1}\right)^2 dx = \pi \int_1^2 \frac{1}{(x+1)^2}\, dx = \pi \int_1^2 (x+1)^{-2}\, dx = \pi\cdot\Big[(-1)\,(x+1)^{-1}\Big]_1^2$$

candidat: $K(x+1)^{-1}$

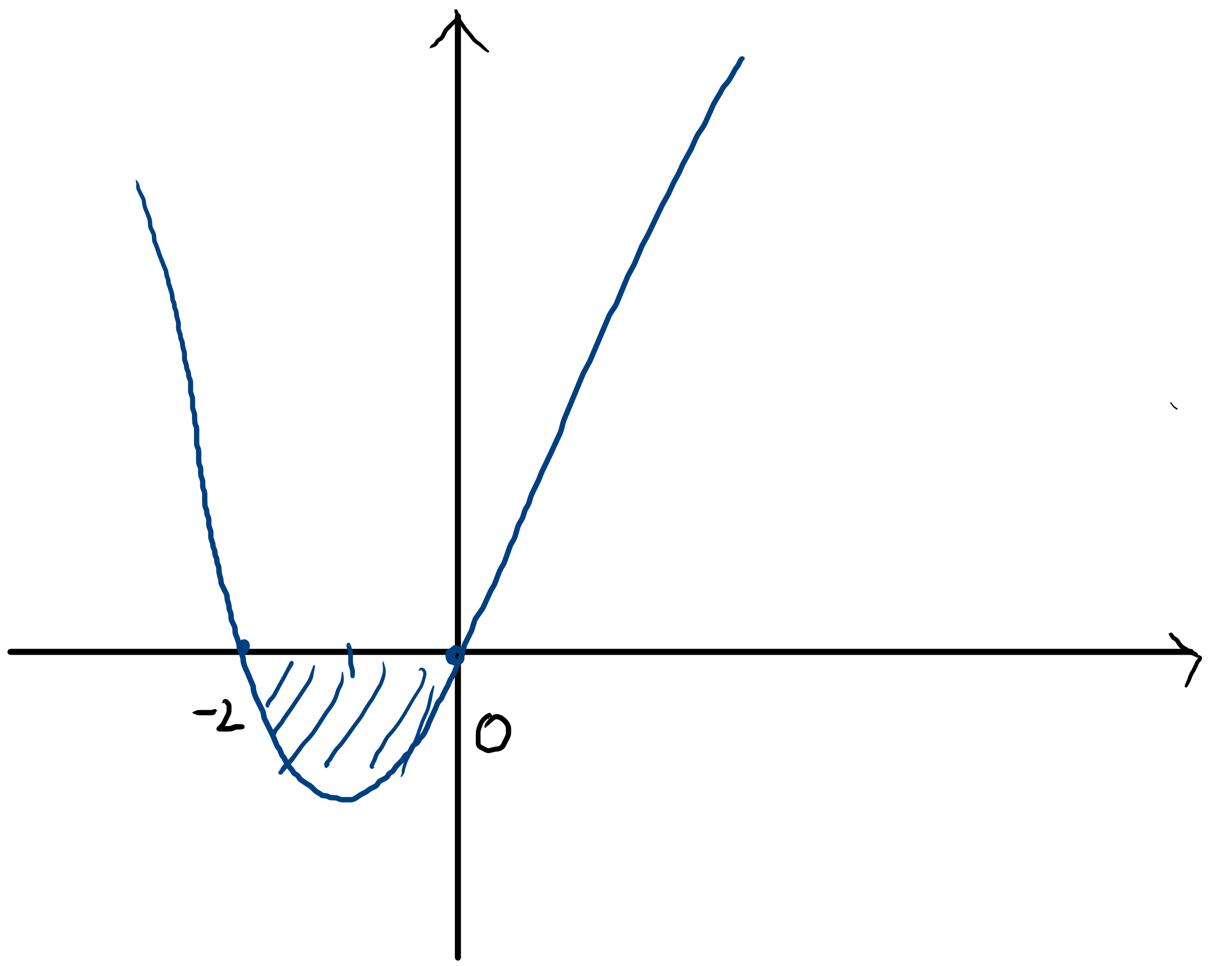
(candidat)' : $\underbrace{-K}\,(x+1)^{-2}$

$K = -1$

$$= \pi\cdot(-1)\left[\frac{1}{x+1}\right]_1^2 = -\pi\left(\frac{1}{3} - \frac{1}{2}\right) = \frac{\pi}{6}$$

1.3.24 Le domaine délimité par la courbe d'équation $y = f(x)$ et l'axe Ox tourne autour de cet axe. Calculer son volume, sachant que :

a) $f(x) = x^2 + 2\,x$ b) $f(x) = \sqrt{1 - x^2}$



$$f(x) = x^2 + 2x = x(x+2)$$

$$V = \pi \int_{-2}^{0} (x^2+2x)^2\, dx$$

$$\int (x^2+2x)^2\, dx = \int (x^4 + 4x^3 + 4x^2)\, dx$$

$$= \frac{1}{5}x^5 + x^4 + \frac{4}{3}x^3 + c$$