15.11.23

1.3.10

1.3.5

1.3.6

i) $f(x) = \ln(\sqrt{3 - x^2})$

$$\left(\ln(u)\right)' = \frac{u'}{u}$$

$$u = \sqrt{3 - x^2} \quad , u' = \frac{-2x}{2\sqrt{3-x^2}} = \frac{-x}{\sqrt{3-x^2}}$$

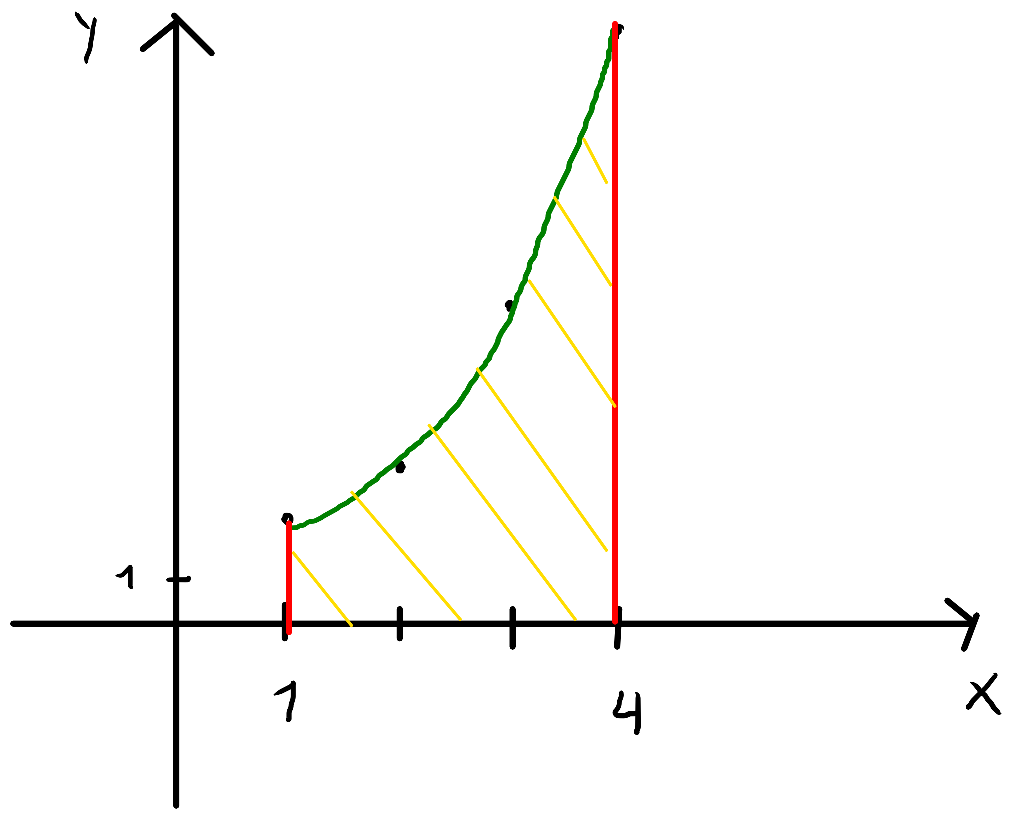
$$f'(x) = \frac{\frac{-x}{\sqrt{3-x^2}}}{\sqrt{3-x^2}} = \frac{-x}{\sqrt{3-x^2}} \cdot \frac{1}{\sqrt{3-x^2}}$$

1.3.10 Calculer :

a) $\displaystyle\int_1^4 (x^2 - 2x + 3)\, dx$

b) $\displaystyle\int_{-1}^1 (2x^3 + 3x^2 + 2x - 1)\, dx$

a)



$f(2) = 4 - 4 + 3 = 3$

$f(3) = 9 - 6 + 3 = 6$

$f(1) = 1 - 2 + 3 = 2$

$f(4) = 16 - 8 + 3 = 11$

$f(x) > 0 \;, \quad \forall x \in [1; 4]$

$$\int (x^2 - 2x + 3)\, dx = \underbrace{\frac{1}{3}x^3 - x^2 + 3x}_{F(x)} + c = F(x) + c$$

$$\int_1^4 (x^2 - 2x + 3)\, dx = F(x)\Big|_1^4 = F(4) - F(1)$$

$$= \left(\frac{1}{3}\cdot 64 - 16 + 12\right) - \left(\frac{1}{3}\cdot 1 - 1 + 3\right) =$$

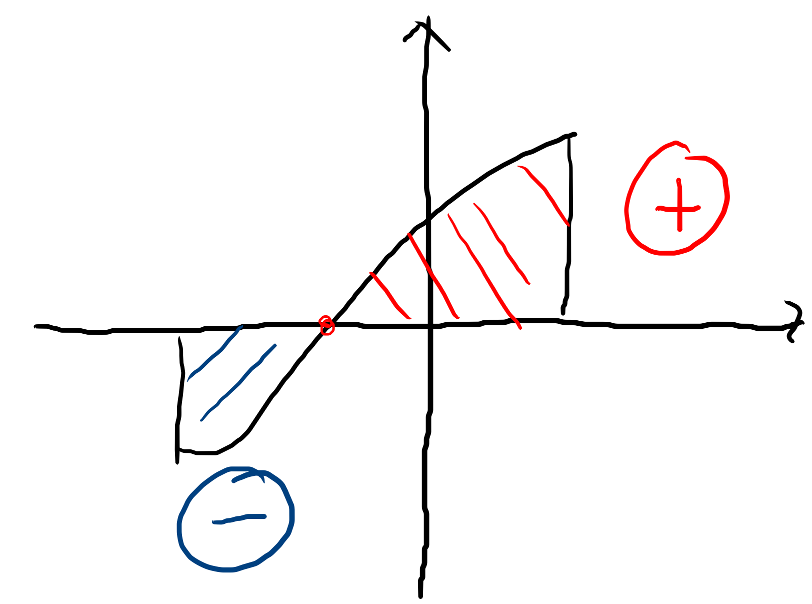
$$= \frac{64}{3} - 4 - \frac{1}{3} - 2 = \frac{63}{3} - 6 = 21 - 6 = 15 \quad [u^2]$$

b) $\int_{-1}^{1} (2x^3 + 3x^2 + 2x - 1)\,dx$

$$\int (2x^3 + 3x^2 + 2x - 1)\,dx = \frac{2}{4}x^4 + \frac{3}{3}x^3 + \frac{2}{2}x^2 - x + C$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C \qquad , n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln(x) + C$$



$$F(x) = \frac{1}{2}x^4 + x^3 + x^2 - x$$

$$\int_{-1}^{1} (2x^3 + 3x^2 + 2x - 1)\,dx = F(x)\Big|_{-1}^{1} = \underbrace{\left(\frac{1}{2} + 1 + 1 - 1\right)}_{F(1)} - \left(\frac{1}{2} \cancel{-1} \cancel{+1} + 1\right)$$

$$= \frac{1}{2} + 1 - \frac{1}{2} - 1 = 0$$