22.09.23

			40
Ma	midi	20	
			40
Me	m		
	s	40	
Je	m		
	s	40	
V	m		
	s	40	140

7x 25 175

1.1.1 Déterminer l'ensemble de définition et la dérivée des fonctions suivantes :

a) $f(x) = e^{5x}$

b) $f(x) = e^{x^2}$

c) $f(x) = e^{1/x}$

d) $f(x) = e^{\sqrt{x^2+x}}$

e) $f(x) = \exp\left(\sqrt{\dfrac{1+x^2}{1-x^2}}\right)$

f) $f(x) = e^{\sin(x)}$

g) $f(x) = x^2\, e^x$

h) $f(x) = e^{-x}\cos(x)$

$$\boxed{(e^u)' = u' \cdot e^u}$$

c) $\left(\frac{1}{x}\right)' = (x^{-1})' = -1 \cdot x^{-2} = \frac{-1}{x^2}$

$\left(e^{\frac{1}{x}}\right)' = -\frac{1}{x^2} \cdot e^{\frac{1}{x}}$

$ED(f) = \mathbb{R}^*$

d) $f(x) = e^{\boxed{\sqrt{x^2+x}}}$

Pour quelles valeurs de x, $\sqrt{x^2+x}$ a-t-elle un sens ?

On détermine le signe de x^2+x.

x		-1		0	
x^2+x	$+$	0	$-$	0	$+$

$x^2 + x = 0$

$x(x+1) = 0$

$0 \quad -1$

$ED(f) =]-\infty ; -1] \cup [0 ; +\infty[$

La dérivée : $\left(\sqrt{x^2+x}\right)' = \dfrac{2x+1}{2\sqrt{x^2+x}}$

$\left(\sqrt{u}\right)' = \dfrac{u'}{2\sqrt{u}}$

$f(x) = \dfrac{2x+1}{2\sqrt{x^2+x}}\, e^{\sqrt{x^2+x}}$

f) $f(x) = e^{\sin(x)}$

g) $f(x) = x^2\, e^x$

h) $f(x) = e^{-x}\, \cos(x)$

$$(\sin(x))' = \cos(x)$$

f) $ED(f) = \mathbb{R}$

$$f'(x) = e^{\sin(x)} \cdot \cos(x)$$

g) $ED(f) = \mathbb{R}$

$$\boxed{\begin{array}{l} u = x^2 \ ; \ u' = 2x \\ v = e^x \ ; \ v' = e^x \end{array}} \quad (uv)' = u'v + u \cdot v'$$

$$f(x) = \underbrace{x^2}_{u} \cdot \underbrace{e^x}_{v}$$

$$f'(x) = 2x\, e^x + x^2\, e^x = x\, e^x (2 + x) = (x+2)\, x\, e^x$$

h) $ED(f) = \mathbb{R}$

$$f(x) = \underbrace{e^{-x}}_{u} \cdot \underbrace{\cos(x)}_{v}$$

$$u = e^{-x} \ ; \ u' = -e^{-x}$$

$$v = \cos(x) \ ; \ v' = -\sin(x)$$

$$f'(x) = -e^{-x} \cdot \cos(x) + e^{-x} \cdot (-\sin(x))$$

$$= -e^{-x} \left[\cos(x) + \sin(x) \right]$$

1.1.2 Calculer la dérivée d'ordre n de $f(x) = \underbrace{x}_{u}\,\underbrace{e^x}_{v}$.

$$f'(x) = \underbrace{(1+x)}_{u}\;\underbrace{e^x}_{v}$$

$$f'(x) = e^x + x \cdot e^x = (1+x)\,e^x$$

$$f''(x) = e^x + (1+x)\,e^x = e^x\left(1 + 1+x\right) = (2+x)\,e^x$$

$$f'''(x) = (3+x)\,e^x$$

$$\vdots$$

$$\boxed{f^{n}(x) = (n+x)\,e^x}$$

1.1.5 Calculer les limites suivantes :

a) $\displaystyle\lim_{x\to 2} \frac{e^x - e^2}{x - 2} \underset{\text{"}\frac{0}{0}\text{"}}{\overset{\text{Ind}}{\overset{\text{BH}}{=}}} \lim_{x \to 2} \frac{e^x}{1} = e^2$

b) $\displaystyle\lim_{x\to 0} \frac{e^x - 1}{\sin(x)} =$

c) $\displaystyle\lim_{x\to 0} \frac{x\, e^x}{1 - e^x} =$

d) $\displaystyle\lim_{\substack{x\to 0 \\ >}} x\, e^{1/x} =$