1.1.4

24.11.23

e) $\displaystyle\int_0^1 x\, e^x \, dx$

f) $\displaystyle\int_1^{\ln(2)} x^2\, e^x \, dx$

Rappel

- $\displaystyle\int e^x dx = e^x + c$
- $\displaystyle\int e^{x^2} dx$ impossible
- $\displaystyle\int x e^{x^2} dx = \frac{1}{2} e^{x^2} + c$

e) $\displaystyle\int x\, e^x \, dx = (x-1)\, e^x + c$

Candidat : $\underbrace{(x+a)}_{u}\underbrace{e^x}_{v}$

$(\text{candidat})'$: $e^x + (x+a)\, e^x = (1 + x + a)\, e^x = \underbrace{(x + a + 1)}_{=x} e^x \Rightarrow a = -1$

$$\int_0^1 x\, e^x \, dx = (x-1)\, e^x \Big|_0^1 = 0 - (-1) = 1$$

f) $\displaystyle\int_1^{\ln(2)} x^2\, e^x\, dx$

$$\int x^2 e^x \, dx = (x^2 - 2x + 2)\, e^x + c$$

candidat : $\underbrace{(x^2 + ax + b)}_{u}\, \underbrace{e^x}_{v}$

(candidat)' : $(2x + a)\, e^x + (x^2 + ax + b)\, e^x = (x^2 + 2x + ax + a + b)\, e^x$

$$= (x^2 + (a+2)\, x + a + b)\, e^x$$

On résout un système d'équations :

$$\begin{cases} a + 2 = 0 \\ a + b = 0 \end{cases} \Leftrightarrow \begin{cases} a = -2 \\ b = 2 \end{cases}$$

$$\left[(x^2 - 2x + 2)\, e^x\right]' = \underbrace{(\cancel{2x} - \cancel{2})}_{0} e^x + (x^2 \cancel{- 2x} + \cancel{2})\, e^x = x^2 e^x$$

$$\int_1^{\ln(2)} x^2 e^x \, dx = (x^2 - 2x + 2)\, e^x \Big|_1^{\ln(2)}$$

$$= \left((\ln(2))^2 - 2\ln(2) + 2\right) e^{\ln(2)} - (1 - 2 + 2)\, e^1$$

$$= (\ln^2(2) - 2\ln(2) + 2) \cdot 2 - e$$

d) $\int_1^2 \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx$

$$\int \frac{1}{\sqrt{x}} e^{-\sqrt{x}}\, dx = -2\, e^{-\sqrt{x}} + c$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

candidat : $K \cdot e^{-\sqrt{x}}$

$$\boxed{\left(e^{u}\right)' = u' \cdot e^{u}}$$

$(\text{candidat})'$: $K \cdot \frac{-1}{2\sqrt{x}}\, e^{-\sqrt{x}} \Rightarrow \frac{-K}{2} = 1 \Rightarrow K = -2$

$$\int_1^2 \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx = -2\, e^{-\sqrt{x}} \Big|_1^2 = -2\left(e^{-\sqrt{2}} - e^{-1}\right)$$

1.1.7 Déterminer les primitives des fonctions suivantes :

$$\int f(x)\,dx = F(x) + c$$

a) $f(x) = \dfrac{1}{x+1}$

b) $f(x) = \dfrac{1}{3x+2}$

$$\boxed{\begin{aligned} &\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c \\ &\int \frac{1}{x}\,dx = \ln(|x|) + c \\ &(\ln(u))' = \frac{u'}{u} \end{aligned}}$$

a) $\displaystyle\int \frac{1}{x+1}\,dx = \ln(|x+1|) + c$

b) $\displaystyle\int \frac{1}{3x+2}\,dx = \frac{1}{3}\ln(|3x+2|) + c$

candidat : $K \cdot \ln(3x+2)$

$(\text{candidat})'$: $\dfrac{3K}{3x+2} \Rightarrow K = \dfrac{1}{3}$

$$\lim_{x \to \infty} \frac{\ln(3x-5)}{6e^{2x}}$$