28.09.23

h) $f(x) = \ln\left(\frac{x^2}{1-x}\right)$

$$[\ln(u)]' = \frac{u'}{u}$$

Condition : $\frac{x^2}{1-x} > 0$

Le signe de $\frac{x^2}{1-x}$:

x		0		1		
x^2	$+$	0	$+$		$+$	0
$1-x$	$+$		$+$		$-$	1
$\frac{x^2}{1-x}$	$+$	0	$+$		$-$	

$$ED(f) =]-\infty, 0[\cup]0,1[$$

$$f'(x) = \frac{\left(\frac{x^2}{1-x}\right)'}{\frac{x^2}{1-x}} = \frac{\frac{2x-x^2}{(1-x)^2}}{\frac{x^2}{1-x}} = \frac{2x-x^2}{(1-x)^{\cancel{2}\,1}} \cdot \frac{\cancel{1-x}\;1}{x^2} = \frac{\cancel{x}(2-x)}{x^{\cancel{2}\,1}(1-x)} = \frac{2-x}{x(1-x)}$$

$$\left(\frac{x^2}{1-x}\right)' = \frac{2x(1-x) - x^2\cdot(-1)}{(1-x)^2} = \frac{2x - 2x^2 + x^2}{(1-x)^2} = \frac{2x - x^2}{(1-x)^2}$$

$u = x^2$; $u' = 2x$

$v = 1-x$; $v' = -1$

i) $f(x) = \ln(\sqrt{3 - x^2})$

j) $f(x) = \ln(3\,x^5)$

i) Signe de $3 - x^2$ ($\frown$)

x		$-\sqrt{3}$		$\sqrt{3}$	
$3 - x^2$	$-$	0	$+$	0	$-$

$ED = \left]-\sqrt{3}\,;\,\sqrt{3}\right[$

$$f'(x) = \frac{\left(\sqrt{3-x^2}\right)'}{\sqrt{3-x^2}} = \frac{\frac{-x}{\sqrt{3-x^2}}}{\sqrt{3-x^2}} = \frac{-x}{\sqrt{3-x^2}} \cdot \frac{1}{\sqrt{3-x^2}} = \frac{-x}{3-x^2} = \frac{x}{x^2-3}$$

$$\left(\sqrt{3-x^2}\right)' = \frac{-\cancel{2}x}{\cancel{2}\sqrt{3-x^2}}$$

$$\boxed{(\sqrt{u})' = \frac{u'}{2\sqrt{u}}}$$

j) $ED(f) = \mathbb{R}_+^*$

$$f'(x) = \frac{(3x^5)'}{3x^5} = \frac{15x^4}{3x^5} = \frac{5}{x}$$

k) $f(x) = x\ln(x) - x$

$ED(f) = \mathbb{R}_+^*$

$$f'(x) = \left(x\ln(x) - x\right)' = \left(x\ln(x)\right)' - (x)'$$

$$= 1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1$$

$$= \ln(x) + 1 - 1 = \ln(x)$$

m) $f(x) = \frac{x}{\ln(x)}$

n) $f(x) = \frac{1}{x\ln(x)}$

m) Deux conditions : • $x > 0$

• $\ln(x) \neq 0 \iff x \neq 1$

$\ln(x) = 0 \iff x = 1$

$ED(f) =]0;1[\cup]1;+\infty[$

Calculons la dérivée :

$u = x \quad ; \quad u' = 1$

$v = \ln(x) \quad ; \quad v' = \frac{1}{x}$

$$f'(x) = \frac{1 \cdot \ln(x) - x \cdot \frac{1}{x}}{(\ln(x))^2} = \frac{\ln(x) - 1}{\ln^2(x)}$$

n) $f(x) = \dfrac{1}{x \ln(x)}$

$$\boxed{\left(\frac{1}{u}\right)' = \frac{-u'}{u}}$$

Comme m) : $ED(f) = \,]0\,;1[\, \cup \,]1\,;+\infty[$

$$f'(x) = \frac{-(x \ln(x))'}{x \ln(x)} = \frac{-\left(\ln(x) + x\,\frac{1}{x}\right)}{x \ln(x)} = \frac{-\left(\ln(x) + \cancel{x}\right)}{x \ln(x)}$$

1.1.13 Étudier les fonctions suivantes :

a) $f(x) = e^{-x^2}$

① Recherche de $ED(f)$

$ED(f) = \mathbb{R}$

② Parité

$f(-x) = e^{-(-x)^2} = e^{-x^2} = f(x)$ $\Rightarrow$ f est paire

$\Rightarrow$ graphique symétrique par rapport à l'axe de y

③ Signe de $f(x)$

x	
$f(x)$	$+$

④ Recherche des AV et AH

$\boxed{a^{-x} = \frac{1}{a^x}}$

AV : aucune

AHD : $\lim\limits_{x \to +\infty} e^{-x^2} = \lim\limits_{x \to +\infty} \frac{1}{e^{x^2}} = 0$

AHG : $\lim\limits_{x \to -\infty} e^{-x^2} = \lim\limits_{x \to -\infty} \frac{1}{e^{x^2}} = 0$

} AH $y = 0$

⑤ Croissance

$f'(x) = (-x^2)' \, e^{-x^2} = -2x \cdot e^{-x^2}$

$\boxed{(e^u)' = u' \cdot e^u}$

Tableau des signes de la dérivée :

x		0	
$-2x$	$+$	0	$-$
e^{-x^2}	$+$		$+$
$f'(x)$	$+$	0	$-$
$f(x)$	$\nearrow$	$\boxed{\text{max}}$	$\searrow$

Tableau de la croissance :

Max $(0 ; 1)$

⑥ Courbure

$$f'(x) = -2x \cdot e^{-x^2}$$

$$f''(x) = -2 \cdot e^{-x^2} + (-2x) \cdot (-2x\, e^{-x^2}) = -2e^{-x^2} + 4x^2 e^{-x^2}$$

$u = -2x \;;\; u' = -2$

$v = e^{-x^2} \;;\; v' = -2x\, e^{-x^2}$

$$= 2e^{-x^2}\left[-1 + 2x^2\right] = 2e^{-x^2}\left(2x^2 - 1\right)$$

Signe de $f''(x)$:

$2x^2 - 1 = 0 \quad \Big| \div 2$

$x^2 - \frac{1}{2} = 0$

$x^2 = \frac{1}{2}$

$x = \pm \frac{1}{\sqrt{2}}$

x		$-1/\sqrt{2}$		$1/\sqrt{2}$	
$2e^{-x^2}$	+		+		+
$2x^2 - 1$	+	0	−	0	+
$f''(x)$	+	0	−		+
courbure $f(x)$		pi		pi	

$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0{,}7$

$$f\left(-\frac{1}{\sqrt{2}}\right) = f\left(\frac{1}{\sqrt{2}}\right) = e^{-1/2} = \frac{1}{\sqrt{e}} \cong 0{,}6$$

7) Graphique

Cloche de Gauss