29.11.23

1.3.11 Sachant que $\int_0^1 f(x)\,dx = 3$, $\int_1^2 f(x)\,dx = 4$ et $\int_2^3 f(x)\,dx = -8$, calculer :

a) $\int_0^2 f(x)\,dx$

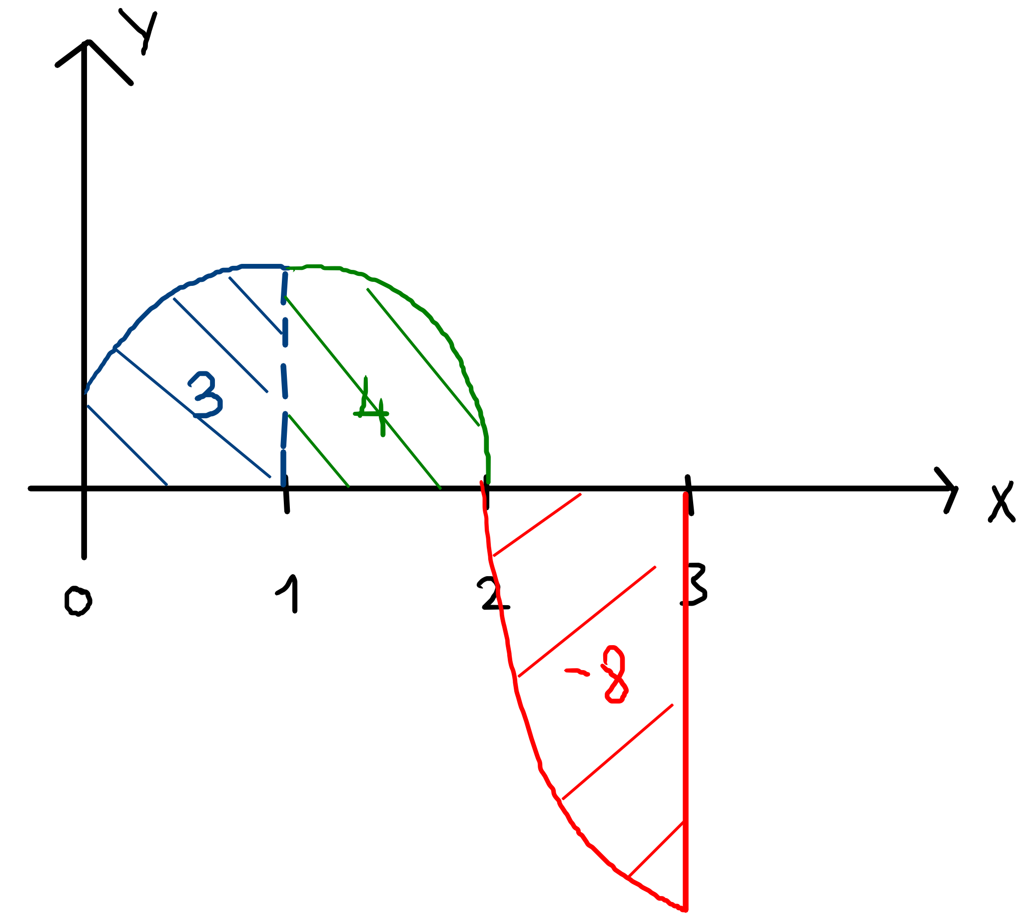
b) $\int_0^1 3f(x)\,dx$

c) $\int_0^3 8f(x)\,dx$

d) $\int_3^1 2f(x)\,dx$

Illustration

Aire algébrique



a) $$\int_0^2 f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx$$

$$= 3 + 4 = 7$$

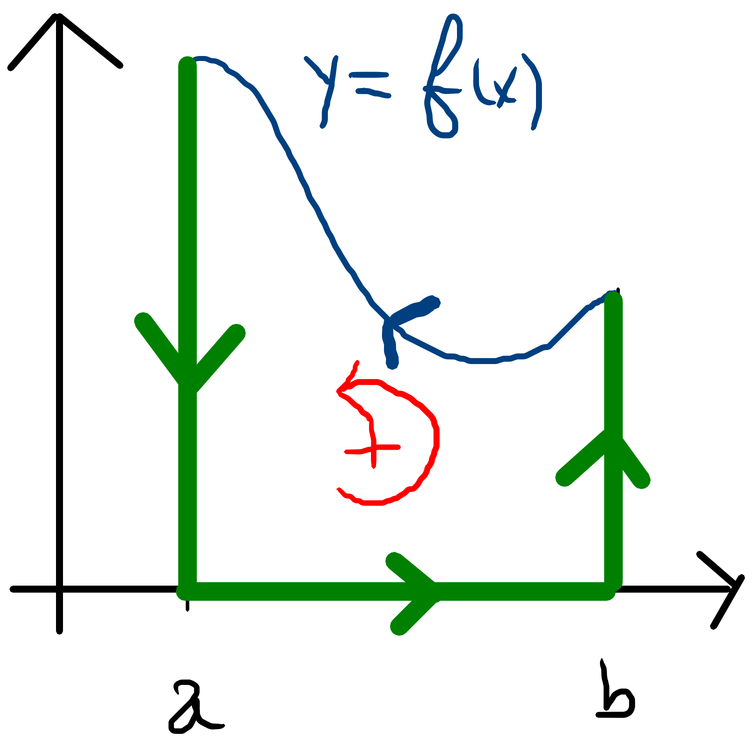
b) $$\int_0^1 3f(x)\,dx = 3\int_0^1 f(x)\,dx = 3\cdot 3 = 9$$

c) $$\int_0^3 8f(x)\,dx = 8\int_0^3 f(x)\,dx = 8\left[\int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx\right]$$

$$= 8\,(3 + 4 - 8) = -8$$

d) $$2\int_3^1 f(x)\,dx = -2\int_1^3 f(x)\,dx = -2\,(4-8) = 8$$

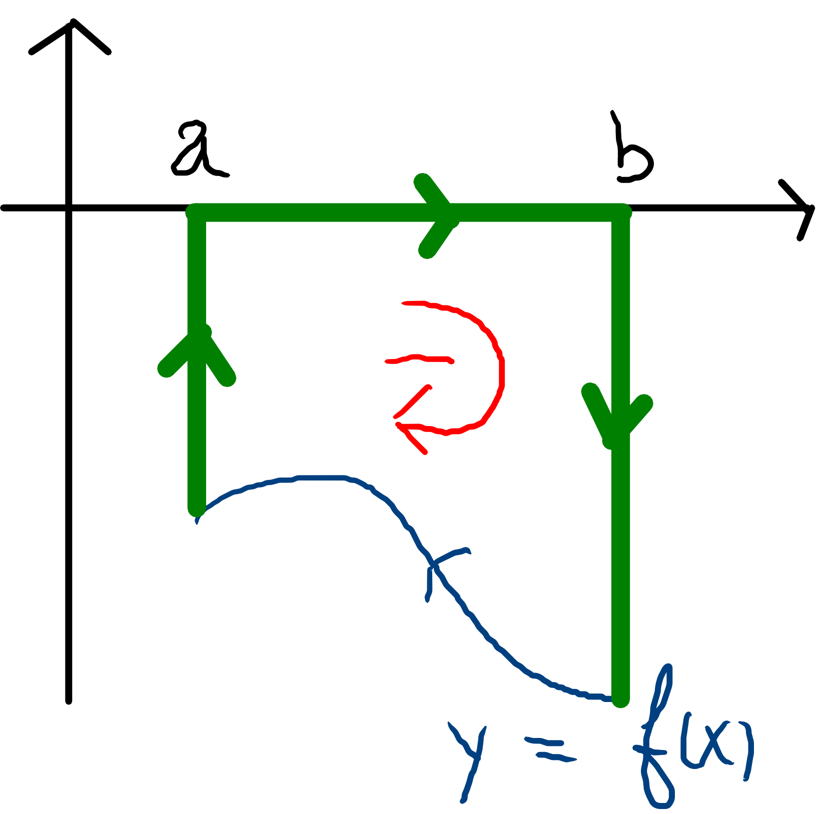
Propriétés des aires algébriques



① $\displaystyle\int_a^b f(x)\,dx > 0$

$a < b$

$\displaystyle\int_b^a f(x)\,dx < 0$



② $\displaystyle\int_a^b f(x)\,dx < 0$

$a < b$

$\displaystyle\int_b^a f(x)\,dx > 0$

Application

Soit $f(x) = x^2 - 2x - 15$.

Calculer :

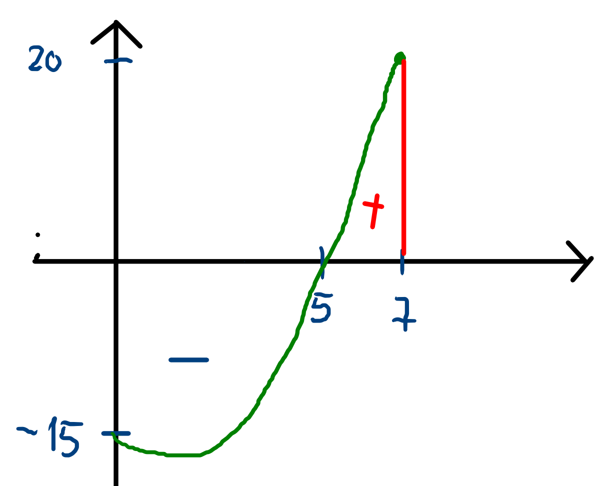
1. L'aire algébrique de $y = f(x)$ entre 0 et 7.
2. L'aire géométrique de $y = f(x)$ entre 0 et 7.

① $$\int_0^7 f(x)\,dx = \int_0^7 \left(x^2 - 2x - 15\right)dx = \left[\frac{1}{3}x^3 - x^2 - 15x\right]_0^7 = \frac{1}{3}\cdot 343 - 49 - 105 =$$
$$= \frac{343 - 147 - 315}{3} = \frac{-119}{3}$$

② $f(x) = (x-5)(x+3)$

Signe de $f(x)$:

x		-3		5	
$f(x)$	$+$	0	$-$	0	$+$



$f(7) = 49 - 14 - 15 = 20$

Aire géométrique : $$\left|\int_0^5 f(x)\,dx\right| + \int_5^7 f(x)\,dx =$$

$$= \left|\left.\frac{x^3}{3} - x^2 - 15x\right|_0^5\right| + \left.\frac{x^3}{3} - x^2 - 15x\right|_5^7 = \frac{175}{3} + \frac{56}{3} = \frac{231}{3}$$

1.3.16