2.3.12 Calculer les intégrales indéfinies :

j) $\displaystyle\int \frac{3x-7}{x^3+x^2+4x+4}\,dx$

Factorisons le dénominateur :

$$x^3 + x^2 + 4x + 4 = x^2(x+1) + 4(x+1) = (x+1)(x^2+4)$$

$$\frac{3x-7}{x^3+x^2+4x+4} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4}$$

$$= \frac{A(x^2+4) + (x+1)(Bx+C)}{(x+1)(x^2+4)}$$

- $x = -1$: $\quad -10 = 5A \quad \Rightarrow \quad A = -2$
- $x = 0$: $\quad -7 = -2 \cdot 4 + C \quad \Rightarrow \quad C = 1$
- $x = 1$: $\quad -4 = -2 \cdot 5 + 2 \cdot (B+1)$

$$6 = 2B + 2 \quad \Rightarrow \quad B = 2$$

$$\frac{3x-7}{x^3+x^2+4x+4} = \frac{-2}{x+1} + \frac{2x+1}{x^2+4}$$

$$\int \frac{3x-7}{x^3+x^2+4x+4}\,dx = \int \frac{-2}{x+1}\,dx + \int \frac{2x+1}{x^2+4}\,dx$$

$$= I_1(x) + I_2(x)$$

$$I_1(x) = -2\ln|x+1| + c_1$$

$$I_2(x) = \int \frac{2x}{x^2+4}\,dx + \int \frac{1}{x^2+4}\,dx$$

$$= \ln|x^2+4| + \frac{1}{2}\arctan\left(\frac{x}{2}\right) + c_2$$

Finalement :

$$\int \frac{3x-7}{x^3+x^2+4x+4}\,dx = \ln(x^2+4) - 2\ln|x+1| + \frac{1}{2}\arctan\left(\frac{x}{2}\right) + c$$

$$= \ln\left(\frac{x^2+4}{(x+1)^2}\right) + \frac{1}{2}\arctan\left(\frac{x}{2}\right) + c$$