

2.3.19

d) $f(x) = \ln\left(\frac{2x}{x+1}\right)$

1) Recherche ED(f): $\frac{2x}{x+1} > 0$

x	-1	0
$\frac{2x}{x+1}$	+	- 0 +

$ED(f) =]-\infty; -1[\cup]0; +\infty[$

2) Pas de parité, ED non symétrique

3) Signe de $f(x)$

$f(x) = 0 \Leftrightarrow \frac{2x}{x+1} = 1 \Leftrightarrow 2x = x+1 \Leftrightarrow x = 1$

x	-1	0	1
$f(x)$	+	///	- 0 +

4) Recherche des asymptotes

$$\underline{AV} : \lim_{\substack{x \rightarrow -1 \\ <}} \ln\left(\frac{2x}{x+1}\right) = +\infty \quad \Rightarrow \text{AVG } x = -\underline{1}$$

$\ln\left(\frac{-2}{0^-}\right)$

$$\lim_{\substack{x \rightarrow 0 \\ >}} \ln\left(\frac{2x}{x+1}\right) = -\infty \quad \Rightarrow \text{AVD } x = 0$$

$\ln\left(\frac{0^+}{1}\right)$

$$\underline{AH} : \lim_{x \rightarrow +\infty} \ln\left(\frac{2x}{x+1}\right) = \ln(2) \quad \Rightarrow \text{AHD} : y = \ln(2)$$

$$\lim_{x \rightarrow -\infty} \ln\left(\frac{2x}{x+1}\right) = \ln(2) \quad \Rightarrow \text{AHG} : y = \ln(2)$$

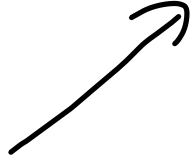
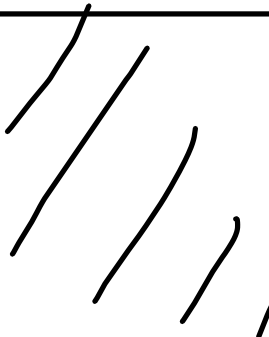
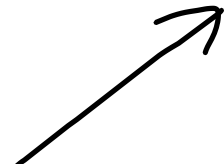
En conclusion $y = \ln(2)$ est une AH.

5) Etude de la croissance

$$\left(\frac{2x}{x+1}\right)' = \frac{2(x+1) - 2x}{(x+1)^2} = \frac{2}{(x+1)^2}$$

$$f'(x) = \frac{x+1}{2x} \cdot \frac{2}{(x+1)^2} = \frac{1}{x(x+1)}$$

$$ED(f') = \mathbb{R}^* - \{-1\}$$

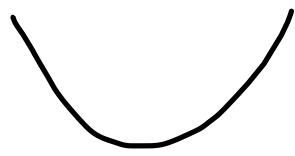
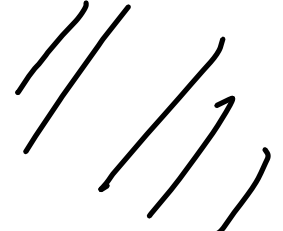
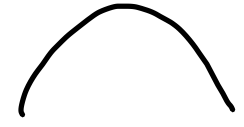
x	-1	0	
$f'(x)$	$+$	$-$	$+$
$f(x)$			

Aucun extrema

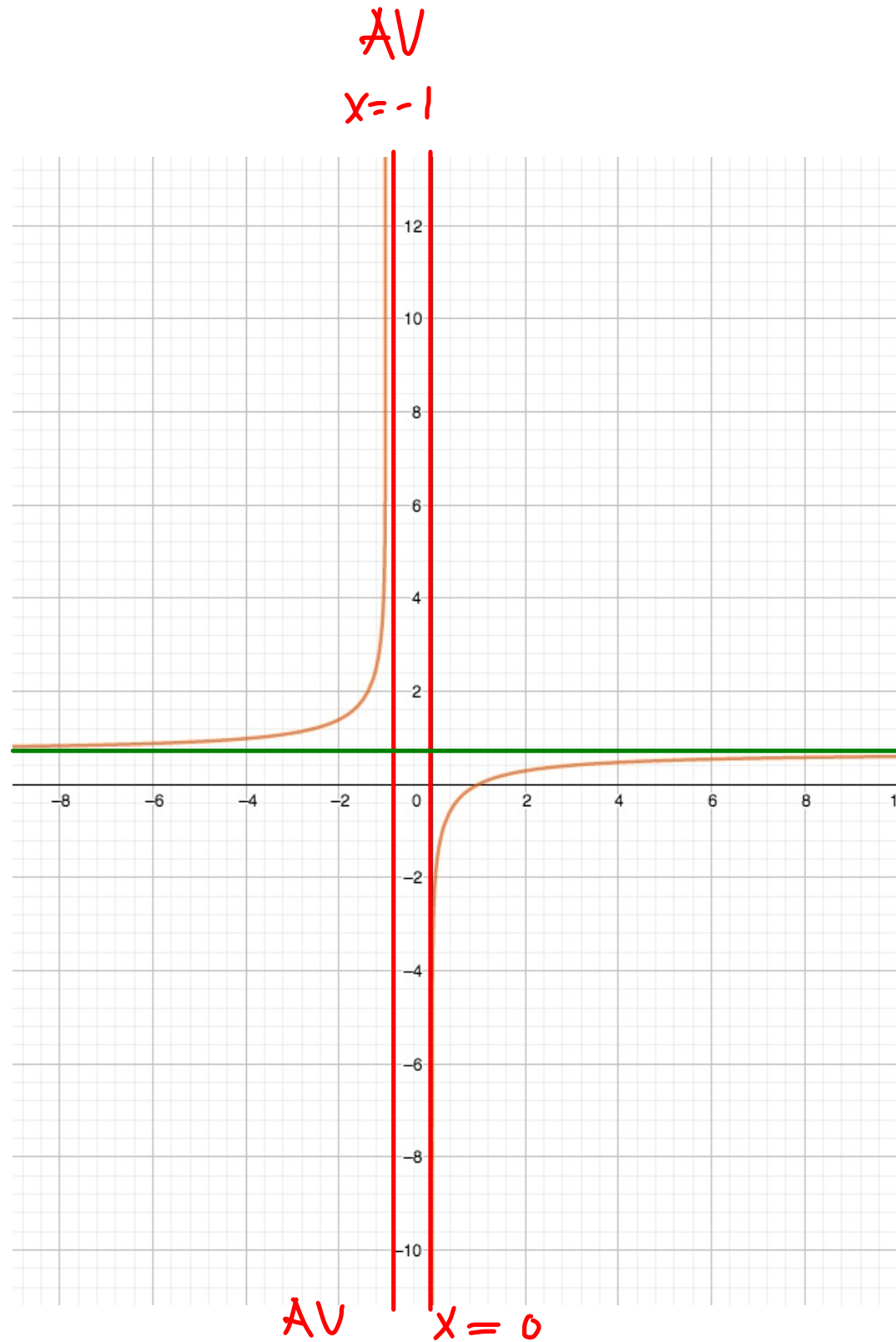
6) Concavité

$$f''(x) = \left(\frac{1}{x^2+x} \right)' = \frac{-(2x+1)}{(x^2+x)^2}$$

$$f''(x) = 0 \Leftrightarrow x = -\frac{1}{2}$$

x	-1	$-\frac{1}{2}$	0
$f''(x)$	$+$	$+$ ϕ $-$	$-$
$f(x)$			

7) Graphique



AH
 $y = \ln(2)$