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2.2.6 Calculer :

c) $\int \cos^3(x)\,dx = \int \cos^2(x)\cdot\cos(x)\,dx = \int (1-\sin^2(x))\cos(x)\,dx$

$= \int \cos(x) - \cos(x)\sin^2(x)\,dx$

$= \int \cos(x)\,dx - \boxed{\int \cos(x)\cdot\sin^2(x)\,dx}$

$= \sin(x) - \boxed{\frac{1}{3}\sin^3(x)} + C$

candidat : $K\cdot\sin^3(x)$

(candidat)' : $3K\sin^2(x)\cdot\cos(x) \Rightarrow K = \frac{1}{3}$

d) $\int \sin^4(x)\,dx = \int (\sin^2(x))^2\,dx = \int \left(\frac{1-\cos(2x)}{2}\right)^2 dx$

CRM : $\sin^2\left(\frac{\alpha}{2}\right) = \frac{1-\cos(\alpha)}{2}$

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$= \int \frac{1-2\cos(2x)+\cos^2(2x)}{4}\,dx = \frac{1}{4}\int 1-2\cos(2x)+\cos^2(x)\,dx$

$= \frac{1}{4}\left[\int 1\cdot dx - \int 2\cos(2x)\,dx + \int \cos^2(2x)\,dx\right]$

$= \frac{1}{4}\left[x - \sin(2x) + \frac{1}{2}x + \frac{1}{8}\sin(4x)\right] + C = \frac{1}{4}\left[\frac{12x - 8\sin(2x) + \sin(4x)}{8}\right]$

$= \frac{12x - 8\sin(2x) + \sin(4x)}{32}$

$\int \cos^2(2x)\,dx = \int \cos^2(u)\,\frac{1}{2}\,du = \frac{1}{2}\int \cos^2(u)\,du$

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$\cos^2(x)$	$\frac{1}{2}(x+\sin(x)\cos(x))$

$= \frac{1}{2}\left(\frac{1}{2}(u+\sin(u)\cos(u))\right)$

$= \frac{1}{2}\left(\frac{1}{2}\left(u+\frac{1}{2}\sin(2u)\right)\right)$

$= \frac{1}{4}\left(2x+\frac{1}{2}\sin(4x)\right)$

$2x = u \Leftrightarrow x = \frac{1}{2}u$

$dx = \frac{1}{2}du$

$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$

2.2.8 Calculer :

a) $\int (3x^2 - 2x + 3)\, dx$

b) $\int \frac{3x^4 - 3x^2 - 7}{4x^2}\, dx$

$$\int ku = k \int u$$
$$\int u + v = \int u + \int v$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1}$$
$$n \neq -1$$

b) $$\int \frac{3x^4}{4x^2} - \frac{3x^2}{4x^2} - \frac{7}{4x^2}\, dx = \frac{3}{4}\int x^2 dx - \frac{3}{4}\int dx - \frac{7}{4}\int x^{-2} dx$$

$$= \frac{1}{4}x^3 - \frac{3}{4}x + \frac{7}{4}x^{-1} + c$$

$$= \frac{1}{4}x^3 - \frac{3}{4}x + \frac{7}{4}\cdot\frac{1}{x} + c = \frac{x^4 - 3x^2 + 7}{4x} + c$$

$$\int x^{-2} dx =$$

candidat : $k\, x^{-1}$

$(\text{candidat})' = -k \cdot x^{-2} \Rightarrow k = -1$