

07.01.25

5) Il semble que $y = \ln(x)$ admette une AO à droite.

Déterminons-la : $y = mx + h$

$$m = \lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \frac{\ln(x)}{x} \quad \text{et} \quad h = \lim_{x \rightarrow +\infty} (f(x) - mx)$$

Par définition $\ln(x) = \int_1^x \frac{1}{t} dt$

$$\text{Si } t > 1, \text{ on a } t > \sqrt{t} \Rightarrow \frac{1}{t} < \frac{1}{\sqrt{t}}$$

$$\text{et pour } x > 1, \text{ on a } \int_1^x \frac{1}{t} dt < \int_1^x \frac{1}{\sqrt{t}} dt$$

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$$\begin{aligned} \int \frac{1}{\sqrt{t}} dt &= \int t^{-\frac{1}{2}} dt = 2t^{\frac{1}{2}} + C \\ &= 2\sqrt{t} + C \end{aligned}$$

$$\ln(x) < 2(\sqrt{x} - 1) < 2\sqrt{x}$$

Finalement $0 < \ln(x) < 2\sqrt{x}$ | $\div x$

$$0 < \frac{\ln(x)}{x} < \frac{2}{\sqrt{x}}$$

Par le théorème des deux gendarmes,

Donc il n'y a pas d'AOD.

$$\lim_{x \rightarrow +\infty} \frac{\ln(x)}{x} = 0$$

2.3.6 Déterminer l'ensemble de définition et la dérivée des fonctions suivantes :

a) $f(x) = \ln(5x)$

x	0
5x	- 0 +

$$ED(f) = \mathbb{R}_+^*$$

$$f'(x) = \frac{1}{5x} \cdot 5 = \frac{1}{x}$$

$$[\ln(u)]' = \frac{u'}{u}$$

f) $f(x) = \ln(\underbrace{x - x^2})$

$$g(x) = x(1-x)$$

x	0	1
$x(1-x)$	- 0 + 0 -	

$$ED(f) =]0; 1[$$

$$f'(x) = \frac{1-2x}{x-x^2}$$

h) $f(x) = \ln\left(\frac{x^2}{1-x}\right)$

Signe de $\frac{x^2}{1-x} = u$:

x	0^P	1^i
$\frac{x^2}{1-x}$	+	+

$$ED(f) =]-\infty; 0[\cup]0; 1[=]-\infty; 1[- \{0\}$$

$$f'(x) = \frac{u'}{u}$$

$$u' = \left(\frac{x^2}{1-x} \right)' = \frac{2x(1-x) - x^2 \cdot (-1)}{(1-x)^2} = \frac{2x - 2x^2 + x^2}{(1-x)^2} = \frac{2x - x^2}{(1-x)^2}$$

$$f'(x) = \frac{2x - x^2}{(1-x)^2} \cdot \frac{1-x}{x^2} = \frac{2x - x^2}{x^2(1-x)}$$

m) $f(x) = \frac{x}{\ln(x)}$ $\ln(x) = 0 \Leftrightarrow x = 1$

n) $f(x) = \frac{1}{x \ln(x)}$ $x \ln(x) = 0 \Leftrightarrow x = 0 \text{ ou } x = 1$

m) $ED(f) = \mathbb{R}_+^* - \{1\} =]0; +\infty[- \{1\}$

$$f'(x) = \left(\frac{x}{\ln(x)} \right)' = \frac{\ln(x) - 1}{\ln^2(x)}$$

$$u = x ; u' = 1$$

$$v = \ln(x) ; v' = \frac{1}{x}$$

n) $ED(f) = \mathbb{R}_+^* - \{1\}$

$$f'(x) = \left(\frac{1}{x \ln(x)} \right)' = \left(\frac{1}{u} \right)' = -\frac{u'}{u^2}$$

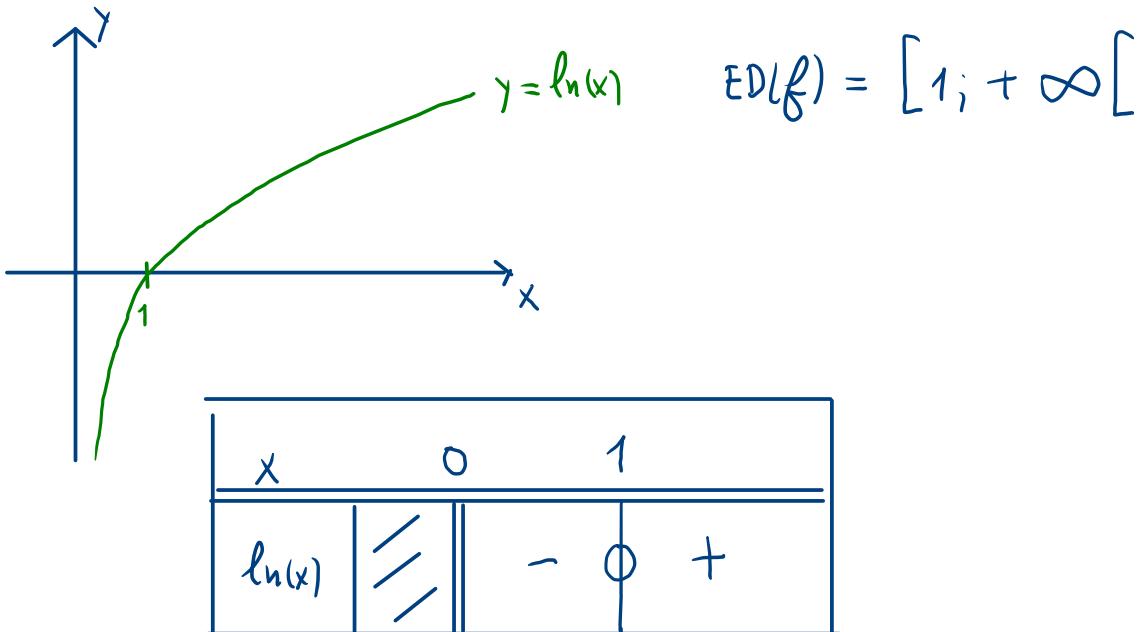
$$= (u^{-1})' = -u^{-2} \cdot u'$$

$$u = x \ln(x)$$

$$u' = \ln(x) + 1$$

$$f'(x) = -\frac{\ln(x) + 1}{(x \ln(x))^2}$$

2.3.7 Calculer les zéros et les extremums de $f(x) = \sqrt{\ln(x)} - \ln(\sqrt{x})$.



$$1) \text{ Zéros de } f(x) : \sqrt{\ln(x)} = \ln(\sqrt{x}), \quad x \geq 1$$

$$\sqrt{\ln(x)} = \ln(x^{\frac{1}{2}})$$

$$\sqrt{\ln(x)} = \frac{1}{2} \ln(x) \quad | \cdot ()^2$$

$$\ln(x) = \frac{1}{4} \ln^2(x)$$

$$\ln^2(x) - 4 \ln(x) = 0$$

$$\ln(x) [\ln(x) - 4] = 0$$

$$\begin{array}{l} \downarrow \\ \ln(x) = 0 \\ \Updownarrow \\ x = 1 \end{array}$$

$$\begin{array}{l} \Downarrow \\ \ln(x) = 4 \\ \Updownarrow \\ x = e^4 \end{array}$$

Les zéros de $f(x)$: 1 et e^4 .

$$2) \quad f(x) = \sqrt{\ln(x)} - \frac{1}{2} \ln(x)$$

$$= \left[\ln(x) \right]^{\frac{1}{2}} - \frac{1}{2} \ln(x)$$

$$f'(x) = \frac{1}{2} \left[\ln(x) \right]^{-\frac{1}{2}} \cdot \left(\ln(x) \right)' - \frac{1}{2} \cdot \frac{1}{x}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{\ln(x)}} \cdot \frac{1}{x} - \frac{1}{2} \cdot \frac{1}{x} = \frac{1}{2x} \left(\frac{1}{\sqrt{\ln(x)}} - 1 \right)$$

$$\text{zéros de la dérivée : } \frac{1}{\sqrt{\ln(x)}} = 1 \Leftrightarrow \sqrt{\ln(x)} = 1 \Leftrightarrow x = e$$

$$\text{Coordonnées : } f(e) = \sqrt{\ln(e)} - \frac{1}{2} \ln(e) = 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow \left(e; \frac{1}{2}\right)$$

Tableau de la croissance :

A hand-drawn graph illustrating the behavior of a function near a vertical asymptote at $x = 1$. The graph is divided into three regions by the asymptote: a left region where $f'(x) > 0$ and $f(x)$ is increasing; a central region where $f'(x) < 0$ and $f(x)$ is decreasing; and a right region where $f'(x) > 0$ and $f(x)$ is increasing again. The word "max" is written in a box above the function curve, with arrows pointing to the local maximum point on the graph.

$$\text{Max} \left(e; \frac{1}{2} \right)$$