12.11.24

2.2.11 Déterminer la fonction f sachant qu'elle admet pour asymptote la droite

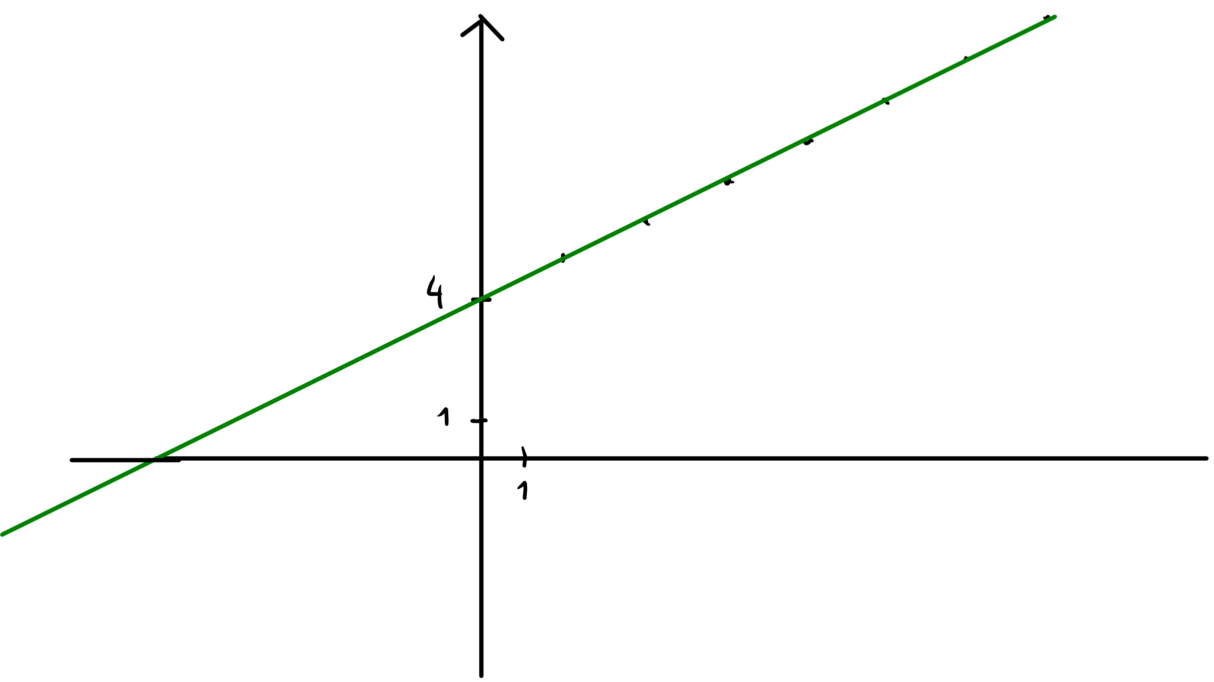
$$x - 2y + 8 = 0$$

et que

$$f''(x) = -\frac{8}{x^3}$$

AO : $x - 2y + 8 = 0$

$$y = \frac{1}{2}x + 4$$



$\Rightarrow c = \frac{1}{2}$ et $d = 4$

$$f''(x) = -8x^{-3}$$

$$f'(x) = 4x^{-2} + c$$

$$f(x) = -4x^{-1} + cx + d$$

$$f(x) = \frac{-4}{x} + cx + d = \frac{cx^2 + dx - 4}{x}$$

$$\boxed{f(x) = cx + d + \frac{-4}{x}}$$

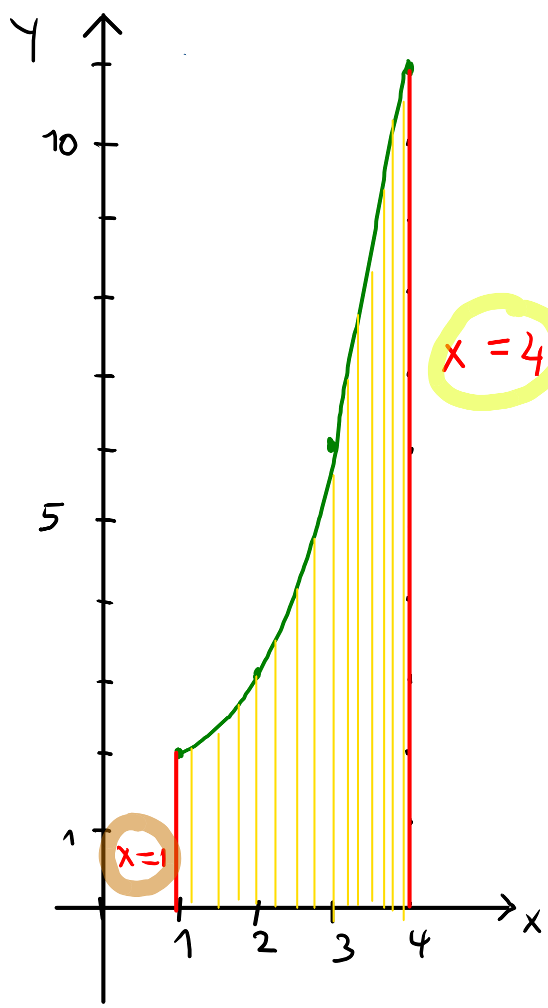
$$f(x) = \text{AO} + \frac{\text{reste}}{\text{diviseur}}$$

2.2.12 Calculer :

a) $\int_1^4 (x^2 - 2x + 3)\, dx$

b) $\int_{-1}^1 (2x^3 + 3x^2 + 2x - 1)\, dx$

Calculer l'aire algébrique entre les droites verticales $x = 1$ $x = 4$, l'axe de x et la courbe $y = x^2 - 2x - 3$



$$f(x) = x^2 - 2x + 3$$

x	$f(x)$
1	2
2	3
3	$9 - 6 + 3 = 6$
4	$16 - 8 + 3 = 11$

$$\int_a^b f(x)\, dx = F(b) - F(a)$$ où F est une primitive de f

$$\int_1^4 x^2 - 2x + 3 = \overbrace{\frac{1}{3}x^3 - x^2 + 3x}^{F(x)} \Big|_1^4$$

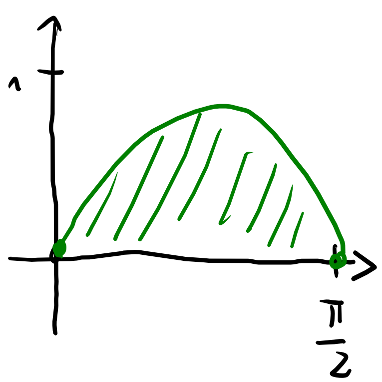
$$= \underbrace{\left(\frac{64}{3} - 16 + 12\right)}_{F(4)} - \underbrace{\left(\frac{1}{3} - 1 + 3\right)}_{F(1)}$$

$$= 21 - 4 - 2 = 15$$

h) $\int_0^{\frac{\pi}{2}} \sin^2(x)\cos(x)\,dx \;=\; \int_0^{\frac{\pi}{2}} (\sin(x))^2 \cdot \cos(x)\,dx \;=\; \frac{1}{3}\sin^3(x)\Big|_0^{\frac{\pi}{2}}$

candidat: $K\cdot(\sin(x))^3 = K\cdot \sin^3(x)$ $\quad = \frac{1}{3}\left(\sin^3\left(\frac{\pi}{2}\right) - \sin^3(0)\right) = \frac{1}{3}$

(candidat)': $K \cdot 3(\sin(x))^2 \cos(x) \Rightarrow 3K = 1 \Rightarrow K = \frac{1}{3}$



e) $\int_0^{\frac{\pi}{4}} (1 + \tan^2(x))\, dx \quad = \quad \tan(x) \Big|_0^{\frac{\pi}{4}} = 1$

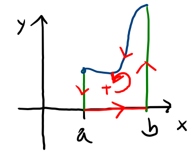
CRM

2.2.14 Montrer que pour une fonction f continue sur $[-a;a]$, on a :

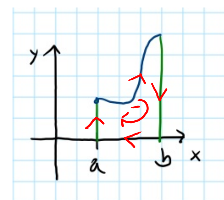
a) $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ lorsque f est paire ;

b) $\int_{-a}^{a} f(x)\,dx = 0$ lorsque f est impaire.

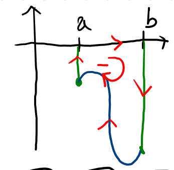
$$\int_a^b f(x)\,dx = F(b) - F(a) = -\big(F(a) - F(b)\big)$$
$$= -\int_b^a f(x)\,dx$$



$\int_a^b > 0$

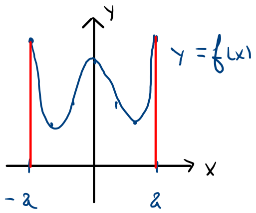


$\int_b^a < 0$



$\int_a^b < 0$

a)



$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$$

paire : $\forall x \in [-a, a]$, $f(x) = f(-x)$

$$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_0^a f(x)\,dx$$
$$= -\int_0^{-a} f(x)\,dx + \int_0^a f(x)\,dx$$

$-x$	t
0	0
$-a$	a

$-x = t \Rightarrow -dx = dt$

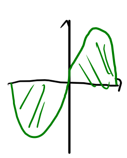
changement de variable

Substitution de x par t

$$= \boxed{-\int_0^a f(-t)\,(-dt)} + \boxed{\int_0^a f(t)\,dt}$$
$$= \int_0^a f(-t)\,dt + \int_0^a f(t)\,dt = \int_0^a f(t)\,dt + \int_0^a f(t)\,dt = 2\int_0^a f(t)\,dt$$

b) $\forall x \in [-a, a]$, $f(-x) = -f(x)$

$$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_0^a f(x)\,dx$$



$$= -\int_0^{-a} f(x)\,dx + \int_0^a f(x)\,dx$$
$$= -\int_0^a f(-x)\,(-dx) + \int_0^a f(x)\,dx$$
$$= \int_0^a f(-x)\,dx + \int_0^a f(x)\,dx$$
$$= -\int_0^a f(x)\,dx + \int_0^a f(x)\,dx = 0$$

2.2.15 Déterminer les réels k pour lesquels on a :

a) $\int_{-1}^{2} kx^2\,dx = \frac{2}{3}$

c) $\int_{0}^{k/2} \cos(t)\,dt = \frac{1}{2}$

a) $K \underbrace{\int_{-1}^{2} x^2\,dx}_{\text{nombre}} = \frac{2}{3}$

$\int_{-1}^{2} x^2\,dx =$