14.11.24

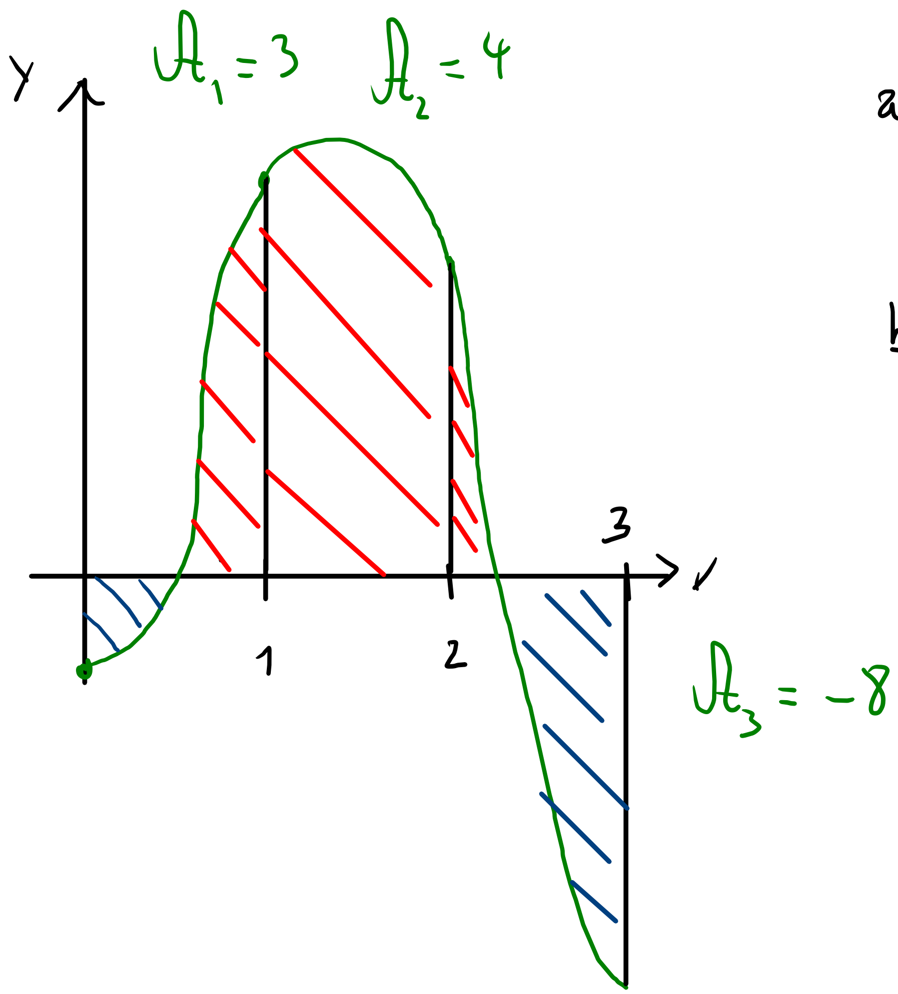
2.2.13 Sachant que $\int_0^1 f(x)\,dx = 3$, $\int_1^2 f(x)\,dx = 4$ et $\int_2^3 f(x)\,dx = -8$, calculer :

a) $\int_0^2 f(x)\,dx$

c) $\int_0^3 8f(x)\,dx$

b) $\int_0^1 3f(x)\,dx$

d) $\int_3^1 2f(x)\,dx$



a) $\int_0^2 f(x)\,dx = \int_0^1 + \int_1^2 = 7$

b) $\int_0^1 3\,f(x) = 3\int_0^1 f(x)\,dx = 9$

c) $8\int_0^3 f(x)\,dx = 8\int_0^1 + 8\int_1^2 + 8\int_2^3 = -8$

d) $\int_3^1 2\,f(x)\,dx = -\int_1^3 2\,f(x)\,dx = -2\left[\int_1^2 + \int_2^3\right] =$

$= -2\cdot(-4) = 8$

2.2.15 Déterminer les réels k pour lesquels on a :

b) $\int_4^k (x^2 - 3x + 7)\,dx = \frac{129}{2}$

$$\left.\frac{1}{3}x^3 - \frac{3}{2}x^2 + 7x\right|_4^k = \left.\frac{2x^3 - 9x^2 + 42x}{6}\right|_4^k = \frac{1}{6}\left[(2k^3 - 9k^2 + 42k) - (128 - 144 + 168)\right]$$

$$= \frac{1}{6}\left(2k^3 - 9k^2 + 42k - 152\right)$$

Donc, on résout l'équation :

$$\frac{1}{6}\left(2k^3 - 9k^2 + 42k - 152\right) = \frac{129}{2} \quad \Big| \cdot 6$$

$$\underbrace{2k^3 - 9k^2 + 42k - 539}_{p} = 0$$

$539 = 7^2 \cdot 11$

$p(7) = 0 \Rightarrow k - 7 \mid p$

Par Horner :

	2	-9	42	-539
7		14	35	539
	2	5	77	0

$$(k-7)(\underbrace{2k^2 + 5k + 77}_{\text{aucun zéro } (\Delta < 0)}) = 0$$

$p(k) = 0 \Leftrightarrow k = 7$

d) $\int_k^0 \frac{2}{(x+1)^3}\,dx = -\int_0^k \frac{3}{(x+3)^2}\,dx$

$$\boxed{\int x^n\,dx = \frac{1}{n+1}\,x^{n+1} + C \qquad n \neq -1}$$

$$-\int_0^k \frac{2}{(x+1)^3}\,dx = -\int_0^k \frac{3}{(x+3)^2}\,dx$$

$$\Rightarrow \int_0^k \frac{2}{(x+1)^3}\,dx = \int_0^k \frac{3}{(x+3)^2}\,dx$$

$$2\int_0^k (x+1)^{-3}\,dx = 2\left(\cdot\frac{1}{-2}(x+1)^{-2}\Big|_0^k\right) = -\left(\frac{1}{(x+1)^2}\Big|_0^k\right) = -\left(\frac{1}{(k+1)^2} - 1\right)$$

$$3\int_0^k (x+3)^{-2}\,dx = 3\left(\frac{1}{-1}(x+3)^{-1}\Big|_0^k\right) = -3\left(\frac{1}{x+3}\Big|_0^k\right) = -3\left(\frac{1}{k+3} - \frac{1}{3}\right)$$

D'où l'équation :

$$1 - \frac{1}{(k+1)^2} = 1 - \frac{3}{k+3}$$

$$\frac{1}{(k+1)^2} = \frac{3}{k+3} \qquad \begin{array}{l} k \neq -1 \\ k \neq -3 \end{array}$$

$$3(k+1)^2 = k+3$$

$$3k^2 + 5k = 0$$

$$3k\left(k + \frac{5}{3}\right) = 0 \iff k = 0 \text{ ou } k = \frac{-5}{3}$$

2.2.16 Déterminer la nature des extremums des fonctions suivantes :

a) $f : x \longmapsto \underbrace{\int_0^x (t^3 - t)\, dt}_{f(x)}$

$f'(x)$ impaire
$f(x)$ paire

$$f'(x) = x^3 - x = x(x^2 - 1) = x(x-1)(x+1)$$

x		-1		0		1	
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$
$f(x)$	$\searrow$	$\boxed{\text{min}}$	$\nearrow$	$\boxed{\text{max}}$	$\searrow$	$\boxed{\text{min}}$	$\nearrow$

Minima : $x = -1$ $\quad (-1 ; -\frac{1}{4})$

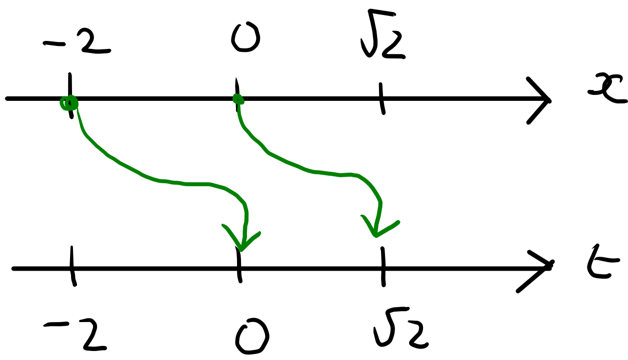
$x = 1$ $\quad (1 ; -\frac{1}{4})$

Maximum $x = 0$ $\quad (0 ; 0)$

$$f(x) = \int_0^x t^3 - t \; dt = \frac{1}{4}t^4 - \frac{1}{2}t^2 \Big|_0^x = \frac{x^4}{4} - \frac{x^2}{2}$$

2.2.17 Calculer les intégrales suivantes à l'aide du changement de variable indiqué :

a) $\displaystyle\int_{-2}^{0} x\sqrt{x+2}\,dx, \quad x = t^2 - 2$



$x = t^2 - 2 \iff t^2 = x + 2 \Rightarrow t = \sqrt{x+2}$

$dx = 2t\,dt$

x	t
-2	0
0	$\sqrt{2}$

① $x = -2 \Rightarrow t^2 = 0 \Rightarrow t = 0$

② $x = 0 \Rightarrow t^2 = 2 \Rightarrow t = \sqrt{2}$ $\quad t > 0$

$f(x) = x\sqrt{x+2}$ $\qquad ED(f) = [-2 ; +\infty[$

$$\int_0^{\sqrt{2}} (t^2-2)\underbrace{\sqrt{t^2}}_{t} \cdot 2t\,dt = 2\int_0^{\sqrt{2}} t^2(t^2-2)\,dt = 2\int_0^{\sqrt{2}} t^4 - 2t^2\,dt$$

$$= 2\left(\frac{t^5}{5} - \frac{2t^3}{3}\Big|_0^{\sqrt{2}}\right) = 2\left(\frac{4\sqrt{2}}{5} - \frac{4\sqrt{2}}{3}\right) = 2\,\frac{12\sqrt{2} - 20\sqrt{2}}{15} = \frac{-16\sqrt{2}}{15}$$

2.2.18 Calculer les intégrales suivantes en effectuant une intégration par parties :

$$(uv)' = u'v + uv' \iff u'v = (uv)' - uv'$$

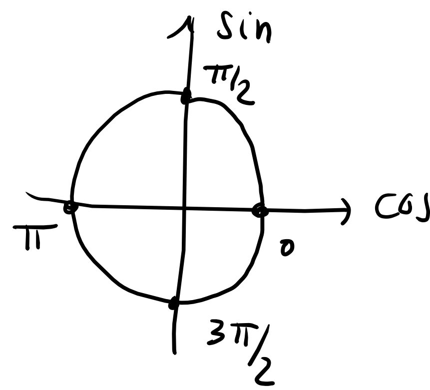
$$\int u'v = \int (uv)' - \int uv'$$

$$\boxed{\int u'v = uv - \int uv'}$$

a) $\displaystyle\int_0^\pi x\sin(x)\,dx = -x\cos(x)\Big|_0^\pi - \int_0^\pi -\cos(x)\,dx = -x\cos(x)\Big|_0^\pi + \int_0^\pi \cos(x)\,dx$

$v = x$ $\qquad v' = dx$

$u' = \sin(x)\,dx$ $\qquad u = -\cos(x)$



$$= -x\cos(x)\Big|_0^\pi + \sin(x)\Big|_0^\pi$$

$$= -(\pi\cos(\pi) - 0) + 0 = \pi$$