2.3.11 Calculer :

a) $\displaystyle\int \frac{dx}{(x+1)^2+4} = \int \frac{du}{u^2+4} = \frac{1}{2}\arctan\left(\frac{u}{2}\right) + c = \frac{1}{2}\arctan\left(\frac{x+1}{2}\right) + c$

CRM

$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)$

Changement de variable $u = x+1 \Leftrightarrow u-1 = x \Leftrightarrow du = dx$

b) $\displaystyle\int \frac{dx}{\underbrace{x^2-4x+8}_{\Delta<0}} = \int \frac{dx}{(x-2)^2+4} = \frac{1}{2}\arctan\left(\frac{x-2}{2}\right) + c$

$x^2 - 4x + 8 = x^2 - 4x + 4 + 4 = (x-2)^2$

CRM

$\frac{1}{x^2+a^2}$	$\frac{1}{a}\arctan\left(\frac{x}{a}\right)$

$$a = \frac{\sqrt{63}}{4} = \frac{3\sqrt{7}}{4}$$

d) $\displaystyle\int \frac{dx}{\underbrace{2x^2-3x+9}_{\Delta<0}} = \frac{1}{2}\int \frac{dx}{x^2-\frac{3}{2}x+\frac{9}{2}} = \frac{1}{2}\int \frac{dx}{(x-\frac{3}{4})^2+\frac{63}{16}} = *$

$$x^2-\frac{3}{2}x+\frac{9}{16} - \frac{9}{16}+\frac{9}{2} = \left(x-\frac{3}{4}\right)^2+\frac{-9+72}{16} = \left(x-\frac{3}{4}\right)^2+\frac{63}{16}$$

On complète les carrés

$$* = \frac{1}{2}\int \frac{dx}{u^2+\left(\frac{3\sqrt{7}}{4}\right)^2} = \frac{1}{\cancel{2}_{1}}\cdot\frac{\cancel{4}^{2}}{3\sqrt{7}}\arctan\left(\frac{4u}{3\sqrt{7}}\right)+c = \frac{2}{3\sqrt{7}}\arctan\left(\frac{4x-3}{3\sqrt{7}}\right)+c$$

changement de variable

$u = x-\frac{3}{4} \Leftrightarrow x = u+\frac{3}{4} \Rightarrow dx = du$

$a = \frac{3\sqrt{7}}{4}$

$$= \frac{2\sqrt{7}}{21}\arctan\left(\frac{4x-3}{3\sqrt{7}}\right)+c$$

CRM

$\frac{1}{\sqrt{r^2-x^2}}$	$\arcsin\left(\frac{x}{r}\right)$

c) $\int \frac{dx}{\sqrt{4x-x^2}} = \int \frac{dx}{\sqrt{4-(x-2)^2}} = \arcsin\left(\frac{x-2}{2}\right) + c$

$4x - x^2 = 4 - (x^2 - 4x + 4)$

$4 - (x-2)^2$

CRM

$\frac{1}{\sqrt{r^2-x^2}}$	$\arcsin\left(\frac{x}{r}\right)$

g) $$\int \frac{2x+3}{\sqrt{9-8x-x^2}}\,dx = \int \frac{2x}{\sqrt{5^2-(x+4)^2}}\,dx + \int \frac{3}{\sqrt{5^2-(x+4)^2}}\,dx = I_1(x) + I_2(x)$$

$$9-(x^2+8x) = 25-(x+4)^2$$

$$I_2(x) = 3\int \frac{dx}{\sqrt{5^2-(x+4)^2}} = 3\cdot \arcsin\left(\frac{x+4}{5}\right) + C_1$$

$$I_1(x) = \int \frac{(2x+8)-8}{\sqrt{5^2-(x+4)^2}}\,dx = \int \frac{2x+8}{\sqrt{5^2-(x+4)^2}}\,dx - 8\cdot\int \frac{dx}{\sqrt{5^2-(x+4)^2}}$$

$$u = (x+4)^2$$

$$du = 2(x+4)\,dx \Rightarrow du = (2x+8)\,dx$$

Il y a plus rapide ...

On fait apparaître au numérateur la dérivée de $9-8x-x^2$

g) $$\int \frac{2x+3}{\sqrt{9-8x-x^2}}\,dx = -\int \frac{-8-2x}{\sqrt{9-8x-x^2}}\,dx + \int \frac{-5}{\sqrt{9-8x-x^2}}\,dx$$

$$= I_1(x) + I_2(x)$$

$$I_1(x) = -\int (-8-2x)(9-8x-x^2)^{-1/2}\,dx = -2\sqrt{9-8x-x^2} + c_1$$

$$u = K(9-8x-x^2)^{1/2}$$

$$u' = \frac{1}{2}K(9-8x-x^2)^{-1/2}(-8-2x) \Rightarrow K=2$$

$$I_2(x) = -5\int \frac{1}{\sqrt{25-(x+4)^2}} = -5\arcsin\left(\frac{x+4}{5}\right) + c_2$$

$$I_1(x) + I_2(x) = \underline{-2\sqrt{9-8x-x^2} - 5\arcsin\left(\frac{x+4}{5}\right) + c}$$