21.11.24

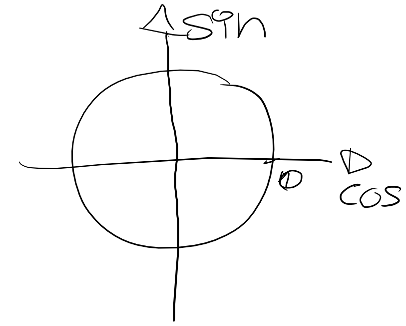
2.2.19 Calculer les intégrales définies suivantes.

b) $\int_0^4 \sqrt{x}(x+2)\,dx$ $= \int_0^4 x^{\frac{1}{2}} \cdot x^1 + 2x^{\frac{1}{2}}\,dx = \int_0^4 x^{\frac{3}{2}}\,dx + 2\int_0^4 x^{\frac{1}{2}}\,dx$

$$= \frac{2}{5}x^{\frac{5}{2}}\Big|_0^4 + \frac{4}{3}x^{\frac{3}{2}}\Big|_0^4 = \frac{2}{5}\cdot\sqrt{x^5}\Big|_0^4 + \frac{4}{3}\sqrt{x^3}\Big|_0^4 =$$

$$\frac{2}{5}\cdot 32 + \frac{4}{3}\cdot 8 = \frac{64}{5} + \frac{32}{3} = \frac{192+160}{15} = \frac{352}{45}$$

c) $\int_0^{\pi/2} \sin^5(x)\cos(x)\,dx = \frac{1}{6}\sin^6(x)\Big|_0^{\pi/2} = \frac{1}{6}\cdot 1 = \frac{1}{6}$



candidat: $F(x) = \sin^6(x)K$

$F'(x) = 6K[\sin(x)]^5 \cdot \cos(x) \Rightarrow K = \frac{1}{6}$

d) $\int_2^{\sqrt{20}} 3x\sqrt{x^2+5}\,dx = \int_2^{\sqrt{20}} 3x(x^2+5)^{\frac{1}{2}}\,dx = (x^2+5)^{\frac{3}{2}}\Big|_2^{\sqrt{20}}$ $\circledast$

candidat: $F(x) = (x^2+5)^{\frac{3}{2}} \cdot K$

$F'(x) = \frac{3}{2}K(x^2+5)^{\frac{1}{2}} \cdot 2x = 3K \cdot x\,(x^2+5)^{\frac{1}{2}} \Rightarrow K = 1$

$\circledast = 125 - 27 = 98$

e) $\int_0^3 \sqrt{9-x^2}\,dx = \int_0^{\frac{\pi}{2}} \sqrt{9-9\sin^2(t)}\;3\cos(t)\,dt = \int_0^{\frac{\pi}{2}} 9\sqrt{1-\sin^2(t)}\,\cos(t)\,dt$

$x = 3\sin(t) \longrightarrow \frac{x}{3} = \sin(t) \quad t = \arcsin(\frac{x}{3})$

$dx = 3\cos(t)\,dt$

x	t
0	0
3	$\frac{\pi}{2}$

$9\int_0^{\frac{\pi}{2}} \sqrt{\cos^2 t}\cdot\cos(t)\,dt = 9\int_0^{\frac{\pi}{2}} \cos^2(t)\,dt = \dots$

$= \frac{9\pi}{4}$

f) $\displaystyle\int_2^3 \frac{5x-2}{x^2-x}\,dx$ element simple

$$\frac{5x-2}{x^2-x} = \frac{5x-2}{x(x-1)} = \frac{a}{x} + \frac{b}{x-1} = \frac{a(x-1)+bx}{x(x-1)} = \frac{ax-a+bx}{x(x-1)}$$

$$= \frac{x(a+b)-a}{x(x-1)} \Rightarrow a = 2 \text{ et } b = 3$$

$$\int_2^3 \frac{2}{x} + \frac{3}{x-1}\,dx = 2\ln(x)\Big|_2^3 + 3\ln(x-1)\Big|_2^3 = 2(\ln(3) - \ln(2)) + 3(\ln(2) - \ln(1))$$

$$= 2\ln(3) - 2\ln(2) + 3\ln(2) - 3\ln(1)$$

$$= 2\ln(3) + \ln(2)$$

$$= \ln(9) + \ln(2) = \ln(18)$$

$$\int_{-1}^0 \frac{dx}{x^2+2x+2} = \int_{-1}^0 \frac{dx}{\underbrace{x^2+2x+1}_{(x+1)^2}+1} = \int_{-1}^0 \frac{dx}{(x+1)^2+1}$$

$t = x+1$

$dt = 1\,dx$

x	t
-1	0
0	1

$$\int \frac{1}{t^2+1}\,dt = \arctan(t)$$

$$= \int_0^1 \frac{dt}{t^2+1} = \arctan(t)\Big|_0^1 = (\pi/4) - 0 = \pi/4$$