31.10.24

2.2.5 Calculer :

a) $\int \cos(3x)\,dx$

b) $\int \sin\left(2x - \frac{\pi}{3}\right)\,dx$

c) $\int (x+3)^3\,dx$

d) $\int (2x-1)^2\,dx$

b) $\int \sin\left(2x - \frac{\pi}{3}\right) dx = -\frac{1}{2}\cos\left(2x - \frac{\pi}{3}\right) + C$

candidat : $F(x) = -K\cos\left(2x - \frac{\pi}{3}\right)$

$F'(x) = -K\left(-\sin\left(2x - \frac{\pi}{3}\right)\cdot 2\right) = 2K\sin\left(2x - \frac{\pi}{3}\right) \Rightarrow 2K = 1$

$\Rightarrow K = \frac{1}{2}$

c) $\int (x+3)^3\,dx = \frac{1}{4}(x+3)^4 + C$

d) $\int (2x-1)^2\,dx = \frac{1}{6}(2x-1)^3 + C$

candidat : $K(2x-1)^3$

(candidat)' : $3K(2x-1)^2 \cdot 2 = 6K(2x-1)^2 \Rightarrow 6K = 1 \Rightarrow K = \frac{1}{6}$

f) $\int (3x^2+x)^3 \overbrace{(6x+1)}^{\text{vient de la dérivée interne}}\,dx = \frac{1}{4}(3x^2+x)^4 + C$

candidat : $K(3x^2+x)^4$

(candidat)' : $K \cdot 4(3x^2+x)^3 \cdot (6x+1) \Rightarrow 4K = 1 \Rightarrow K = \frac{1}{4}$

$\left[(\sin(x))^3\right]' = 3(\sin(x))^2 \cdot \cos(x) = 3\sin^2(x)\cos(x)$

h) $\int \sin^2(x)\cos(x)\,dx = \int (\sin(x))^2 \cdot \cos(x)\,dx = \frac{1}{3}\sin^3(x) + C$

candidat : $K(\sin(x))^3$

$(\text{candidat})'$: $K \cdot 3(\sin(x))^2 \cdot \cos(x) = 3K(\sin(x))^2\cos(x) \Rightarrow K = \frac{1}{3}$

i) $\int \frac{\tan^2(x)}{\cos^2(x)}\,dx = \int \tan^2(x) \cdot \frac{1}{\cos^2(x)}\,dx = \frac{1}{3}\tan^3(x) + C$

candidat : $K \cdot \tan^3(x) = K(\tan(x))^3$

$(\text{candidat})'$: $K \; 3 \cdot (\tan(x))^2 \cdot \frac{1}{\cos^2(x)} \Rightarrow K = \frac{1}{3}$

CRM

$f(x)$	$f'(x)$
$\tan(x)$	$\frac{1}{\cos^2(x)} = 1 + \tan^2(x)$

j) $\int \sqrt{x+3}\,dx = \int (x+3)^{\frac{1}{2}}\,dx = \frac{2}{3}(x+3)^{\frac{3}{2}} + C = \frac{2}{3}\sqrt{(x+3)^3} + C$

k) $\int \frac{dx}{\sqrt{3x+1}} = \int (3x+1)^{-\frac{1}{2}}\,dx = \frac{2}{3}(3x+1)^{-\frac{1}{2}} + C = \frac{2}{3}(3x+1)^{\frac{1}{2}} + C$

$= \frac{2}{3}\sqrt{3x+1} + C$

candidat : $K(3x+1)^{\frac{1}{2}}$

(candidat)' : $K \cdot \frac{1}{2}(3x+1)^{-\frac{1}{2}} \cdot 3 = K \cdot \frac{3}{2}(3x+1)^{-\frac{1}{2}} \Rightarrow K \cdot \frac{3}{2} = 1$

$\Rightarrow K = \frac{2}{3}$

l) $\int \frac{x+1}{\sqrt{x^2+2x}}\,dx = \int (x^2+2x)^{-\frac{1}{2}}(x+1)\,dx = \sqrt{x^2+2x} + C$

candidat : $K(x^2+2x)^{\frac{1}{2}}$

(candidat)' : $K \cdot \cancel{\frac{1}{2}}(x^2+2x)^{-\frac{1}{2}} \cdot \underbrace{(2x+2)}_{\cancel{2}(x+1)} = K(x^2+2x)^{-\frac{1}{2}}(x+1) \Rightarrow K = 1$

2.2.6 Calculer :

b) $\int \sin^2(x)\,dx$ $= \int \frac{1-\cos(2x)}{2}\,dx = \frac{1}{2}\int 1-\cos(2x)\,dx$

$= \frac{1}{2}\left[x - \frac{1}{2}\sin(2x)\right] + C$

$= \frac{1}{2}\left[x - \frac{1}{2}\cdot 2\sin(x)\cos(x)\right] + C$

$= \frac{1}{2}\left(x - \sin(x)\cos(x)\right) + C$

CRM pg 30

$$\sin^2\left(\frac{\alpha}{2}\right) = \frac{1-\cos(\alpha)}{2}$$

CRM pg 31

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$

CRM pg 81

$f(x)$	$F(x)$
$\sin^2(x)$	$\frac{1}{2}(x - \sin(x)\cos(x))$