Division euclidienne dans $\mathbb{N}$

06.11.24

Divisons 2024 par 11.

$$\begin{array}{r|ll}
\overset{\text{dividende}}{\overline{20}24} & \overset{\text{diviseur}}{11} & \\ \hline
-11 & 1 & \\
92 & & \\
-88 & 8 & \text{quotient} \\
44 & & \\
-44 & 4 & \\
\text{reste } 0 & &
\end{array}$$

$$2024 = 11 \cdot 184 + 0$$

11 divise 2024. On écrit $11 \mid 2024$

Pour tout $a, b \in \mathbb{N}$, il existe $q, r \in \mathbb{N}$

$$\begin{cases} a = q \cdot b + r \\ r < b \end{cases}$$

Divisions de polynômes

$$5x \cdot \left(3x^2 - 2x + 1\right) = 15x^3 - 10x^2 + 5x$$

$$\frac{15x^3 - 10x^2 + 5x}{5x} = 3x^2 - 2x + 1$$

$$\begin{array}{rrrr|l}
 & 15x^3 & -10x^2 & +5x & 5x \\ \cline{5-5}
- & 15x^3 & & & 3x^2 \\ \cline{2-2}
 & 0 & -10x^2 & +5x & -2x \\
 & - & -10x^2 & & +1 \\ \cline{3-3}
 & & & 5x & \\
 & & - & 5x & \\ \cline{4-4}
 & & & 0 &
\end{array}$$

Divisons $x^3 + x^2 + x + 1$ par $x - 1$

$$\begin{array}{rrrrr|l}
 & x^3 & +x^2 & +x & +1 & x - 1 \\ \cline{6-6}
- & x^3 & -x^2 & & & x^2 + 2x + 3 \\ \cline{2-3}
 & & 2x^2 & +x & +1 & \\
- & & 2x^2 & -2x & & \\ \cline{3-4}
 & & & 3x & +1 & \\
 & & & 3x & -3 & \\ \cline{4-5}
\end{array}$$

reste : 4

Preuve :

$$\boxed{(x-1)(x^2+2x+3) + 4} = x^3+x^2+x+1$$

$$= x^3+2x^2+3x-x^2-2x-3+4$$

$$= x^3+x^2+x+1 \quad ✓$$

$$\begin{array}{rl|l}
 & x^4+x^3+x^2+x+1 & x+1 \\ \hline
- & \underline{x^4+x^3} & x^3+x \\
 & \quad\quad x^2+x+1 & \\
 & \quad\quad \underline{x^2+x} & \\
\end{array}$$

reste : 1

$$x^4+x^3+x^2+x+1 = (x+1)(x^3+x) + 1$$

Diviser 1) x^3+x^2+x+1 par x^2+1

2) x^3+2x^2+3 par x^2-x-1

$$\begin{array}{rl|l}
1) & x^3+x^2+x+1 & x^2+1 \\ \hline
- & \underline{x^3 \quad\quad +x} & x+1 \\
 & \quad x^2 \quad\quad +1 & \\
 & \quad \underline{x^2 \quad\quad +1} & \\
\end{array}$$

reste 0

$$x^3+x^2+x+1 = (x+1)(x^2+1)$$

2)

$$\begin{array}{r|l} x^3 + 2x^2 \ \ldots \ +3 & x^2 - x - 1 \\ \hline - \quad x^3 - x^2 - x & x \\ \hline 3x^2 + x + 3 & +3 \\ - \quad 3x^2 - 3x - 3 & \\ \hline \text{reste} \quad 4x + 6 & \end{array}$$

$$x^3+2x^2+3 = (x+3)(x^2-x-1) + (4x+6)$$

2.3.1 → 2.3.8