2.4 Fractions rationnelles

21.11.24

$\frac{\text{polynômes en } x, y, z, t}{\text{Un autre polynômes en } x, y, z, t}$

Ex $\quad \frac{3x-1}{5y-2}, \quad \frac{3xy+z}{5xy-z}$

2.4.1 Rendre les fractions rationnelles irréductibles :

a) $\frac{54a^3b^3}{15a^5b^2}$ b) $\frac{-16u^2v^2w^3}{-4u^3vw^2}$ c) $\frac{x-1}{2x-2}$

a) $\frac{\cancel{3}\cdot 18\,\cancel{a^3}\cdot \cancel{b^2}\cdot b}{\cancel{3}\cdot 5\,\cancel{a^3}\cdot a^2\cdot \cancel{b^2}} = \frac{18b}{5a^2}$

b) $\frac{-16u^2v^2w^3}{-4u^3vw^2} = \frac{-16}{-4}\cdot\frac{u^2}{u^3}\cdot\frac{v^2}{v}\cdot\frac{w^3}{w^2} = \frac{4}{1}\,\frac{1}{u}\,\frac{v}{1}\cdot\frac{w}{1} = \frac{4vw}{u}$

c) $\frac{x-1}{2x-2} = \frac{\cancel{x-1}}{2(\cancel{x-1})} = \frac{1}{2}$

$$\boxed{\begin{array}{l} \frac{a-b}{b-a} = \frac{a-b}{-a+b} = \frac{\cancel{a-b}}{-(\cancel{a-b})} = \frac{1}{-1} = -1 \\ \frac{x^2-2x+1}{-x^2+2x-1} = -1 \end{array}}$$

d) $\frac{2x-2y}{3y-3x} = \frac{2(\cancel{x-y})}{-3(\cancel{x-y})} = -\frac{2}{3}$

e) $\dfrac{a^2-b^2}{(a-b)^2} = \dfrac{\cancel{(a-b)}(a+b)}{(a-b)^{\cancel{2}\,1}} = \dfrac{a+b}{a-b}$

g) $\dfrac{x-x^3}{x^4+2x^3+x^2} = \dfrac{-x^3+x}{x^2(x^2+2x+1)} = \dfrac{-x\cdot(x^2-1)}{x^2(x+1)^2} =$

$= \dfrac{-\cancel{x}(x-1)\cancel{(x+1)}}{x^{\cancel{2}\,1}(x+1)^{\cancel{2}\,1}} = \dfrac{-(x-1)}{x(x+1}$

h) $\dfrac{3z^2-21z+36}{2z^2-12z+18} = \dfrac{3\cdot(z^2-7z+12)}{2(z^2-6z+9)} = \dfrac{3\cancel{(z-3)}(z-4)}{2\cancel{(z-3)}(z-3)}$

$= \dfrac{3(z-4)}{2(z-3)}$

l) $\dfrac{6x^2+2x}{27x^3+1} = \dfrac{2x\cdot\cancel{(3x+1)}}{\cancel{(3x+1)}(9x^2-3x+1)} = \dfrac{2x}{9x^2-3x+1}$

$(3x)^3+1^3$

$\boxed{A^3+B^3=(A+B)(A^2-AB+B^2)}$

page 7

m) $\dfrac{1-x^2+x^3-x^5}{x+x^2-x^3-x^4} = \dfrac{-x^5+x^3-x^2+1}{-x^4-x^3+x^2+x} = \dfrac{\cancel{-}(x^5-x^3+x^2-1)}{\cancel{-}(x^4+x^3-x^2-x)}$

$$= \frac{x^3(x^2-1)+1\cdot(x^2-1)}{x^3(x+1)-x(x+1)} = \frac{(x^2-1)(x^3+1)}{(x+1)(x^3-x)}$$

$$= \frac{(x-1)(x+1)(x+1)(x^2-x+1)}{(x+1)\,x\,(x^2-1)} = \frac{\cancel{(x-1)}\cancel{(x+1)}\cancel{(x+1)}(x^2-x+1)}{\cancel{(x+1)}\,x\,\cancel{(x-1)}\cancel{(x+1)}}$$

$$= \frac{x^2-x+1}{x}$$

o) $\dfrac{2x^3+9x^2+7x-6}{2x^3+x^2-13x+6} = \dfrac{(x+2)(2x^2+5x-3)}{(x-2)(2x^2+5x-3)} = \dfrac{(x+2)\cancel{(2x-1)}\cancel{(x+3)}}{(x-2)\cancel{(2x-1)}\cancel{(x+3)}}$

$P = 2x^3+9x^2+7x-6$

$P(-2) = -16+36-14-6 = 0 \Rightarrow x+2 \,/\, P$

Par Horner :

	2	9	7	−6
−2		−4	−10	6
	2	5	−3	0

$$= \frac{x+2}{x-2}$$

$P = 2x^3+x^2-13x+6$

$P(2) = 16+4-26+6 = 0 \Rightarrow x-2 \,/\, P$

Par Horner

	2	1	−13	6
2		4	10	−6
	2	5	−3	0