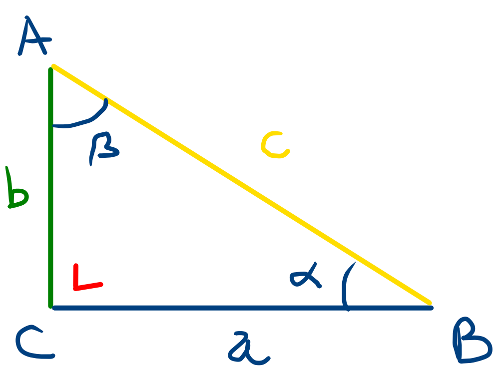
Trigo dans le triangle rectangle

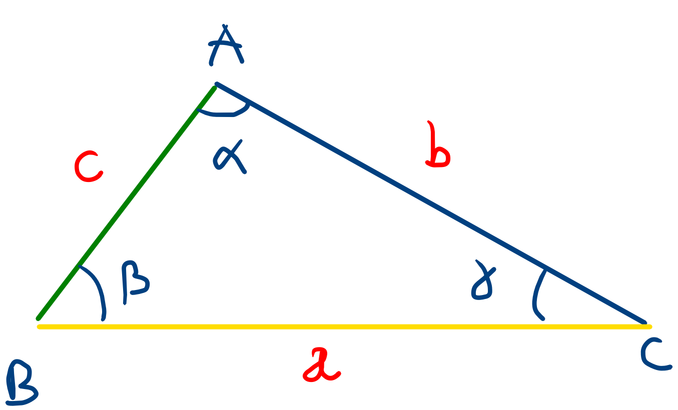


$$\sin(\alpha) = \frac{b}{c}$$

$$\cos(\alpha) = \frac{a}{c}$$

$$\tan(\alpha) = \frac{b}{a}$$

Trigo dans le triangle quelconque

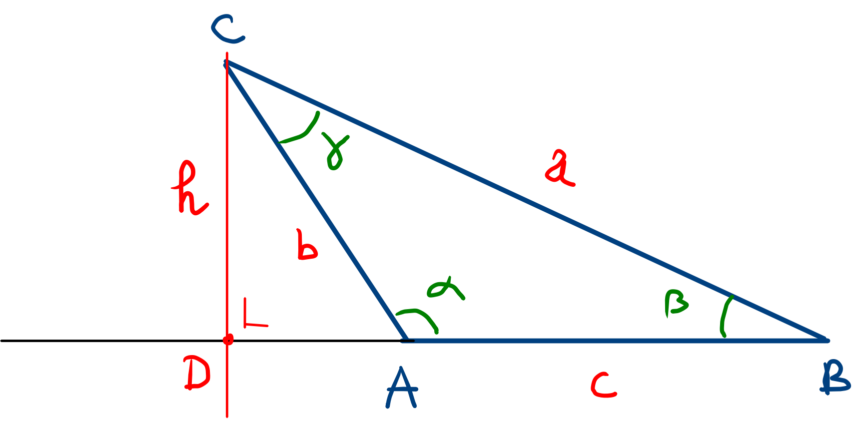


(1) Théorème du cosinus

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos(\alpha)}$$

② Théorème du sinus

$$3 = \frac{6}{2}$$

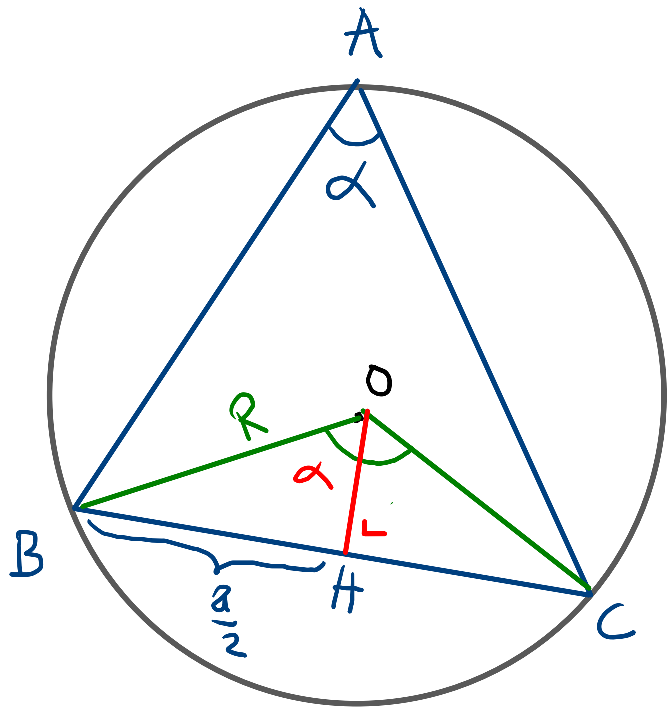


- $\sin(\beta) = \dfrac{h}{a} \Rightarrow h = \sin(\beta) \cdot a$
- $\widehat{DAC} = 180° - \alpha$
- $\sin(180° - \alpha) = \sin(\alpha) = \dfrac{h}{b} \Rightarrow h = \sin(\alpha) \cdot b$

Donc $\sin(\beta) \cdot a = \sin(\alpha) \cdot b \Rightarrow \dfrac{\sin(\beta)}{b} = \dfrac{\sin(\alpha)}{a}$

$$\boxed{\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}}$$

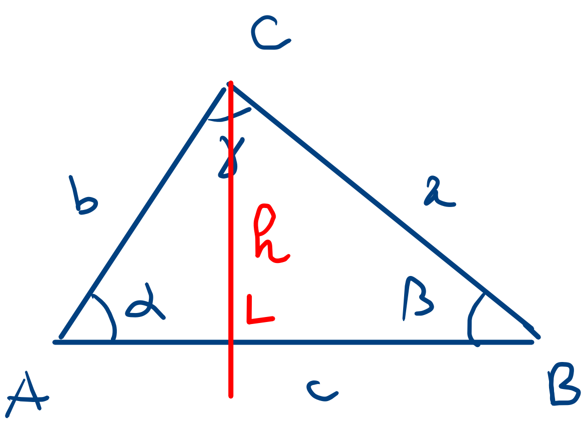
$2^{\text{ème}}$ démonstration



- $2 \cdot \widehat{BAC} = \widehat{BOC}$
- $\widehat{BAC} = \widehat{BOH}$
- $BH = \frac{1}{2} BC = \frac{1}{2} a$
- $\sin(\alpha) = \dfrac{\frac{a}{2}}{R} \Rightarrow R = \dfrac{\frac{a}{2}}{\sin(\alpha)} \Rightarrow 2R = \dfrac{a}{\sin(\alpha)}$

Donc $$\boxed{2R = \frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}}$$

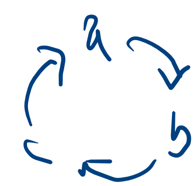
Théorème de l'aire



$$\text{Aire } \Delta ABC = \frac{1}{2} \underbrace{\text{base}}_{c} \cdot \underbrace{\text{hauteur}}_{h}$$

$$\sin(\alpha) = \frac{h}{b} \Rightarrow h = b \sin(\alpha)$$

$$\mathcal{A} = \frac{1}{2} c \cdot b \sin(\alpha) = \frac{1}{2} b \cdot c \cdot \sin(\alpha)$$

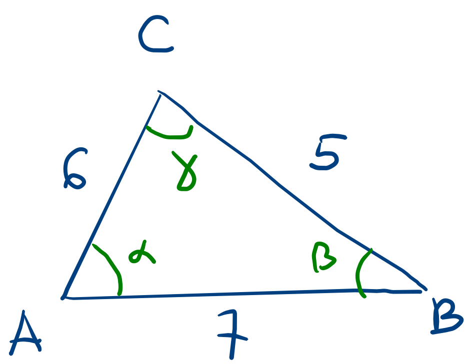


$$\boxed{\mathcal{A} = \frac{1}{2} bc \sin(\alpha) = \frac{1}{2} ac \sin(\beta) = \frac{1}{2} ab \sin(\gamma)}$$

4.4.4 Chaque ligne du tableau ci-dessous donne des informations sur un triangle ABC quelconque On sait que l'on a mesuré les longueurs en centimètres et les angles en degrés.

	a	b	c	α	β	γ	$\mathcal{A}$
a)	5	6	7				
b)	5	7		35			
c)	4	9				54	
d)			8	40	80		
e)	6	5					12
f)		4		70			10

a)



Théorème du cosinus

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

$$25 = 36 + 49 - 84 \cdot \cos(\alpha)$$

$$25 - 36 - 49 = -84 \cdot \cos(\alpha)$$

$$-60 = -84 \cos(\alpha)$$

$$\cos(\alpha) = \frac{60}{84} \xRightarrow{\boxed{TI}} \alpha \cong 44{,}42^\circ$$

$$\Rightarrow \alpha = \begin{cases} 44{,}42^\circ + k \cdot 360^\circ \\ -44{,}42^\circ + k \cdot 360^\circ \end{cases}$$