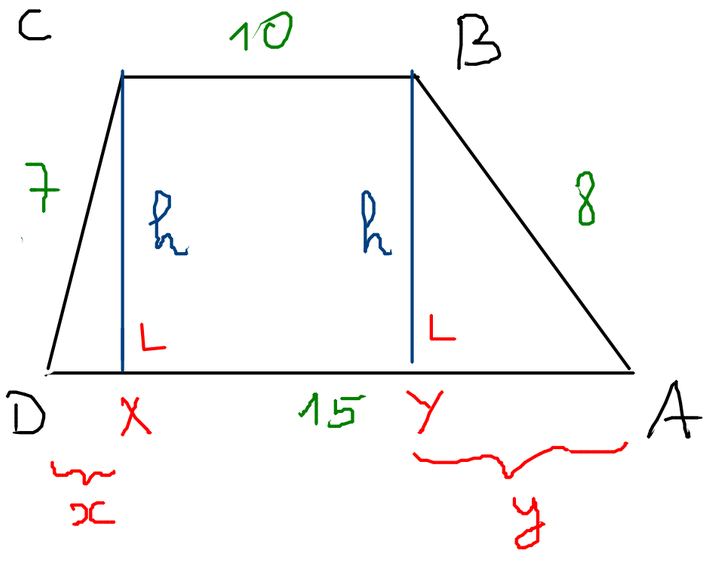
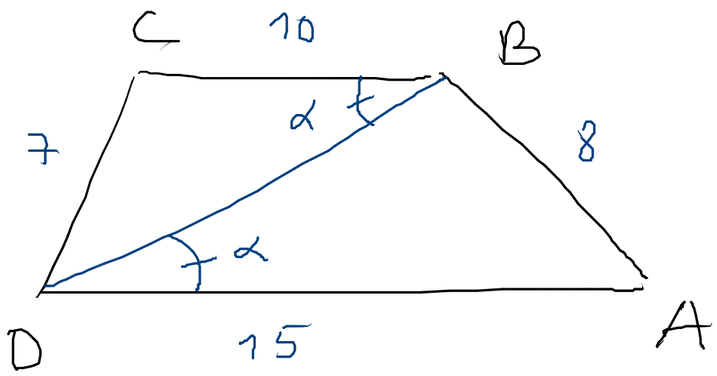
4.4.15 Dans le trapèze $ABCD$, les bases sont $\overline{AD} = 15$ m, $\overline{BC} = 10$ m et les côtés non parallèles sont $\overline{AB} = 8$ m, $\overline{CD} = 7$ m. Calculer les angles et l'aire du trapèze.



$$\begin{cases} x + y = 5 \\ 7^2 - x^2 = h^2 \\ 8^2 - y^2 = h^2 \end{cases}$$

$$\begin{cases} x + y = 5 \\ 49 - x^2 = 64 - y^2 \\ h^2 = 49 - x^2 \end{cases}$$



$BD = x$

$$\begin{cases} 8^2 = 15^2 + BD^2 - 2 \cdot 15 \cdot BD \cdot \cos(\alpha) \\ 7^2 = 10^2 + BD^2 - 2 \cdot 10 \cdot BD \cdot \cos(\alpha) \end{cases}$$

$$\begin{cases} 64 - 225 = x^2 - 30 \cos(\alpha)\, x \\ 49 - 100 = x^2 - 20 \cos(\alpha)\, x \end{cases} \Rightarrow \cos(\alpha) = \frac{161 + x^2}{30x}$$

$$-51 = x^2 - 20x \frac{161 + x^2}{30x} \Rightarrow 3x^2 - 2(x^2 + 161) + 153 = 0$$

$$x^2 = 169 \Rightarrow \underline{x = 13}$$

$$\widehat{DCB} = \arccos \frac{7^2 + 10^2 - 13^2}{2 \cdot 7 \cdot 10} \simeq 98{,}21°$$

$$\widehat{BAD} = \arccos \frac{15^2 + 8^2 - 13^2}{2 \cdot 15 \cdot 8} = 60°$$

$$\widehat{ADC} \simeq 81{,}79°$$

$$\widehat{ABC} = 120°$$