Ex 2.6.3 $\qquad D = \mathbb{R}$

a) $\frac{1}{m} - x = \frac{x}{m} + m \;\Big|\cdot m \qquad V = \mathbb{R}^*$

$1 - mx = x + m^2$

$mx + x = 1 - m^2$

$(m+1)x = (1-m)(1+m)$

(i) $m \neq -1$: $\quad x = 1 - m \qquad S_m = \{1 - m\}$

(ii) $m = -1$:

$1 + x = 1 + x \qquad S_{-1} = \mathbb{R}$

b) $\frac{x - 2a}{a - 1} + \frac{x}{a(1-a)} = \frac{1 - 2a}{a}$

$V = \mathbb{R}^* - \{1\}$

$a(x - 2a) - x = (1 - 2a)(a - 1)$

$ax - \cancel{2a^2} - x = a - 1 - \cancel{2a^2} + 2a$

$(a - 1)x = 3a - 1$

$S_a = \left\{\frac{3a - 1}{a - 1}\right\}$

c) $\frac{x}{n-1} + \frac{x}{n+1} = \frac{1}{(n+1)(n-1)}$

$$V = \mathbb{R} - \{\pm 1\}$$

$$(n+1)x + (n-1)x = 1$$

$$2nx = 1$$

(i) $n \neq 0$: $\quad S = \left\{ \dfrac{1}{2n} \right\}$

(ii) $n = 0$: $\quad S_0 = \emptyset$

d) $\quad x - 1 - \dfrac{x}{b+1} = \dfrac{1}{b-1}$

$$V = \mathbb{R} - \{\pm 1\}$$

$$(b^2-1)(x-1) - (b-1)x = b+1$$

$$b^2x - \cancel{x} - b^2 \cancel{+1} - bx \cancel{+x} = b \cancel{+1}$$

$$b^2x - bx = b^2 + b$$

$$(b^2 - b)x = b^2 + b$$

$$x = \frac{\cancel{b}(b+1)}{\cancel{b}(b-1)}$$

$$x = \frac{b+1}{b-1} \qquad S = \left\{ \frac{b+1}{b-1} \right\}$$

e) $\quad V = \mathbb{R}^*$

$$x - m - 4 + 3x + 3m = mx + m$$
$$4x - mx = -m + 4$$
$$(4-m)x = -m + 4$$

(i) $m \neq 4$: $x = 1$ $\qquad S' = \{1\}$

(ii) $m = 4$: $\qquad S'_4 = \mathbb{R}$

f) $V = \mathbb{R}^*$

$$3(x-y) - 6y = 2(y - 2x) + xy$$
$$3x - 3y - 6y = 2y - 4x + xy$$
$$7x - xy = 11y$$
$$(7-y)x = 11y$$

(i) $y \neq 7$: $x = \dfrac{11y}{7-y}$ $\qquad S = \left\{\dfrac{11y}{7-y}\right\}$

(ii) $y = 7$: $0 = 77$ $\qquad S_7 = \emptyset$

g) $V = \mathbb{R}^*$

$$y(x+2) - (1 - xy) = xy^2$$
$$xy + 2y - 1 + xy = xy^2$$
$$2xy - xy^2 = 1 - 2y$$
$$(2y - y^2)x = 1 - 2y$$

$$(y^2-2y)x = 2y-1$$

$$y(y-2)x = 2y-1$$

(i) $y \neq 2$: $\quad x = \dfrac{2y-1}{y(y-2)}$

$$S = \left\{ \frac{2y-1}{y(y-2)} \right\}$$

(ii) $y = 2$: $\quad S_2 = \emptyset$

h) $V = \mathbb{R} - \{\pm 1\}$

$$(x-2)(z+1) + 2x(z-1) = 3(z-1)(z+1)$$

$$xz + x - 2z - 2 + 2xz - 2x = 3z^2 - 3$$

$$3xz - x = 3z^2 + 2z - 1$$

$$(3z-1)x = 3z^2 + 2z - 1$$

$$(3z-1)x = (3z-1)(z+1)$$

(i) $x \neq \frac{1}{3}$: $\cancel{(3z-1)}\,x = \cancel{(3z-1)}(z+1)$

$$x = z+1$$

$$S = \{z+1\}$$

(ii) $x = \frac{1}{3}$: $\quad S = \mathbb{R}$

i) $\dfrac{x+1}{a-1} - \dfrac{x-1}{a} = \dfrac{ax-1}{a(a-1)}$

$V = \mathbb{R}^* - \{1\}$

$a(x+1) - (a-1)(x-1) = ax - 1$

$\cancel{ax} + a - \cancel{ax} + a + x \cancel{-1} = ax \cancel{-1}$

$x - ax = -2a$

$ax - x = 2a$

$(a-1)x = 2a$

$x = \dfrac{2a}{a-1}$ $\qquad S = \left\{\dfrac{2a}{a-1}\right\}$

j) $\dfrac{x+1}{(b-1)(b-2)} + \dfrac{2b(3-b)}{(b-2)(b-1)(b+1)} = \dfrac{x-1}{(b-2)(b+1)}$

$V = \mathbb{R} - \{-1; 1; 2\}$

$(x+1)(b+1) + 2b(3-b) = (x-1)(b-1)$

$\cancel{xb} + x + b \cancel{+1} + 6b - 2b^2 = \cancel{xb} - x - b \cancel{+1}$

$x + x = -b - 7b + 2b^2$

$2x = 2b^2 - 8b$

$x = b^2 - 4b$

$S = \{b^2 - 4b\}$

k) $V = \mathbb{R}^* - \{-5; 5\}$

$(x+a)a(a+5) - (a-x)(a-5)(a+5) = (x-a)a(a-5)$

$+2a \quad a(a+5)$

$$\cancel{a^2x}+5ax+a^3+5a^2-(a-x)(a^2-25)=\cancel{a^2x}-5ax-a^3-5a^2$$
$$+2a^3+10a^2$$

$$5ax+\cancel{a^3}+\cancel{5a^2}-a^3+25a+a^2x-25x=\cancel{a^3}+\cancel{5a^2}-5ax$$
$$+10a^2$$

$$10ax+a^2x-25x=a^3-25a+10a^2$$

$$(a^2+10a-25)x=a(a^2+10-25)$$

(i) $\underbrace{a^2+10a-25}\neq 0$

$$\Delta=200=(10\sqrt{2})^2 \qquad a=\frac{-10\pm10\sqrt{2}}{2}$$

$a\notin\{-5\pm5\sqrt{2}\}$: $x=a$ $\qquad S=\{a\}$

(ii) $a^2+10a-25=0$: $\qquad S=\mathbb{R}$

l) $V=\mathbb{R}^*-\{\pm1\}$

$$\frac{x-a}{a+1}-\frac{x+a}{a}=\frac{x-1}{a-1}-3$$

$$(x-a)a(a-1)-(x+a)(a+1)(a-1)=(x-1)a(a+1)$$
$$-3a(a+1)(a-1)$$

...

$$(a^2+2a-1)x=a(a^2+2a-1)$$

(i) $a^2+2a-1\neq0$: $x=a$ $\qquad S=\{a\}$

(ii) $a^2+2a-1=0$: $\qquad S=\mathbb{R}$