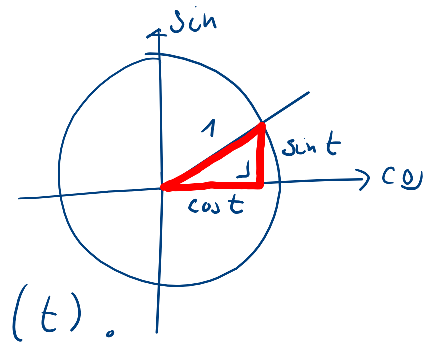
06.03.26

4.3.5 Résoudre les équations suivantes.

d) $3\sin^2(t) + \cos^2(t) - 2 = 0$



$\sin^2(t) + \cos^2(t) = 1$

Nous substituons $\sin^2(t)$ par $1-\cos^2(t)$.

$$3\left(1-\cos^2(t)\right) + \cos^2(t) - 2 = 0$$

$$\Leftrightarrow \quad 3 - 3\cos^2(t) + \cos^2(t) - 2 = 0$$

$$\Leftrightarrow \quad -2\cos^2(t) + 1 = 0$$

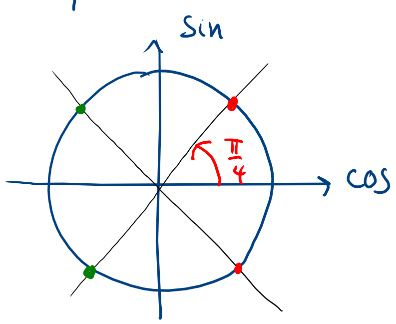
$$\Leftrightarrow \quad \cos^2(t) = \frac{1}{2}$$

$$\cos(t) = \pm\sqrt{\frac{1}{2}}$$

$$\left[\ \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}\right.$$

① $\cos(t) = \frac{\sqrt{2}}{2}$

$$\left[\begin{array}{l} t = \frac{\pi}{4} + k \cdot 2\pi \\ \text{ou} \\ t = -\frac{\pi}{4} + k \cdot 2\pi \end{array}\right. \qquad k \in \mathbb{Z}$$



② $\cos(t) = -\frac{\sqrt{2}}{2}$

$$\left[\begin{array}{l} t = \frac{3\pi}{4} + k \cdot 2\pi \\ \text{ou} \\ t = -\frac{3\pi}{4} + k \cdot 2\pi \end{array}\right. \qquad k \in \mathbb{Z}$$

Finalement, la solution s'écrit

$$\underline{t = \frac{\pi}{4} + k \cdot \frac{\pi}{2}, \quad k \in \mathbb{Z}}$$

g) $8\cos^2(t)+5\sin(t)-1=0$

$$\sin^2(\alpha)+\cos^2(\alpha)=1$$

$$8\left(1-\sin^2(t)\right)+5\sin(t)-1=0$$

$$8-8\sin^2(t)+5\sin(t)-1=0$$

$$-8\sin^2(t)+5\sin(t)+7=0$$

$$8\sin^2(t)-5\sin(t)-7=0$$

Posons $x=\sin(t)$. On résout

$$8x^2-5x-7=0 \qquad -1\leq x\leq 1$$

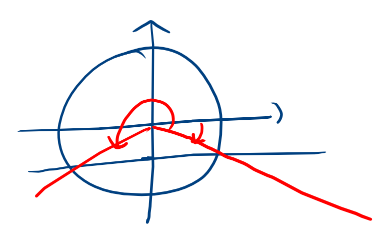
$$\Delta = 249$$

$$x_1=\frac{5+\sqrt{249}}{16}\cong 1{,}29873$$ valeur impossible

$$x_2=\frac{5-\sqrt{249}}{16}\cong -0{,}67373$$

$$\Rightarrow \sin(t)\cong -0{,}67373 \overset{\boxed{TI}}{\Rightarrow} t\cong -42{,}36°$$

$$\begin{cases} t=-42{,}36°+K\cdot 360° \\ t=222{,}36°+K\cdot 360° \end{cases}$$



4.3.6 Résoudre les équations suivantes.

a) $3\cos(x) + 2\sin(x) = -3$

$$2\sin(x) = -3 - 3\cos(x)$$

$$\sin(x) = -\frac{3}{2} - \frac{3}{2}\cos(x)$$

Nous substituons cette valeur du $\sin(x)$ dans l'expression $\sin^2(x) + \cos^2(x) = 1$

$$\begin{cases} \sin(x) = -\frac{3}{2} - \frac{3}{2}\cos(x) \\ \sin^2(x) + \cos^2(x) = 1 \end{cases}$$

$$\left(-\frac{3}{2} - \frac{3}{2}\cos(x)\right)^2 + \cos^2(x) = 1$$

Posons $z = \cos(x)$, $\quad -1 \leq z \leq 1$

$$\left(-\frac{3}{2} - \frac{3}{2}z\right)^2 + z^2 = 1$$

$$\frac{9}{4} + \frac{9}{2}z + \frac{9}{4}z^2 + z^2 - 1 = 0 \quad \Big| \cdot 4$$

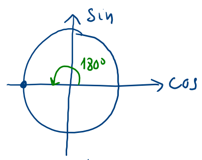
$$9 + 18z + 9z^2 + 4z^2 - 4 = 0$$

$$13z^2 + 18z + 5 = 0$$

$$(13z + 5)(z + 1) = 0$$

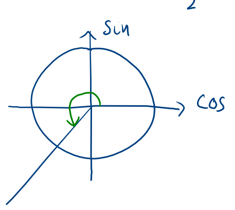
On obtient ② $z = \frac{-5}{13}$ ou ① $z = -1$

① $\begin{cases} \cos(x) = -1 \\ \sin(x) = 0 \end{cases}$ de la substitution



Ce système me donne la solution $x = 180° + k \cdot 360°$

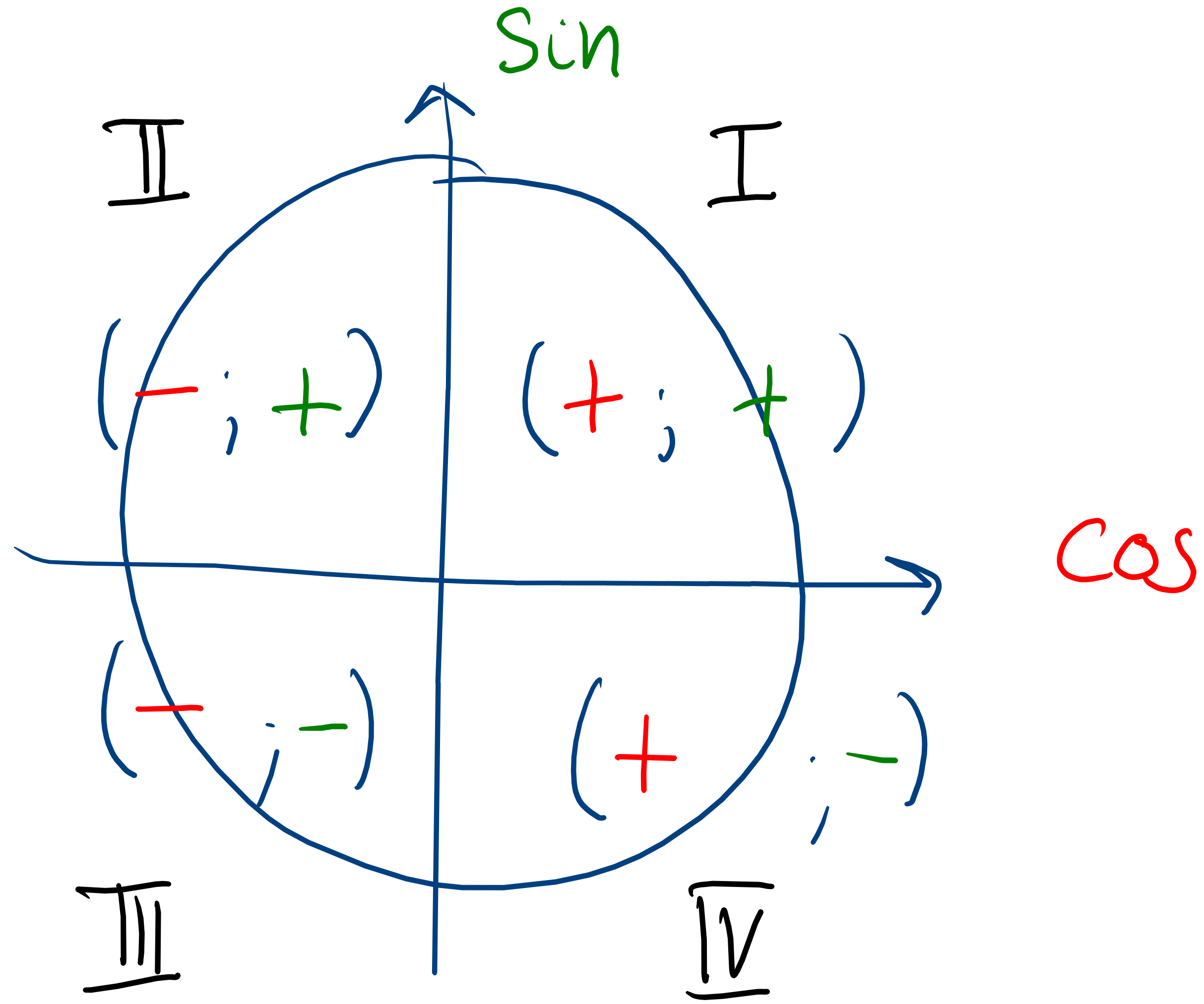
② $\begin{cases} \cos(x) = \frac{-5}{13} \\ \sin(x) = -\frac{3}{2} - \frac{3}{2} \cdot \left(\frac{-5}{13}\right) = -\frac{3}{2} + \frac{15}{26} = \frac{-39 + 15}{26} = \frac{-24}{26} = \frac{-12}{13} \end{cases}$



$x \cong -112.61986495° + k \cdot 360°$

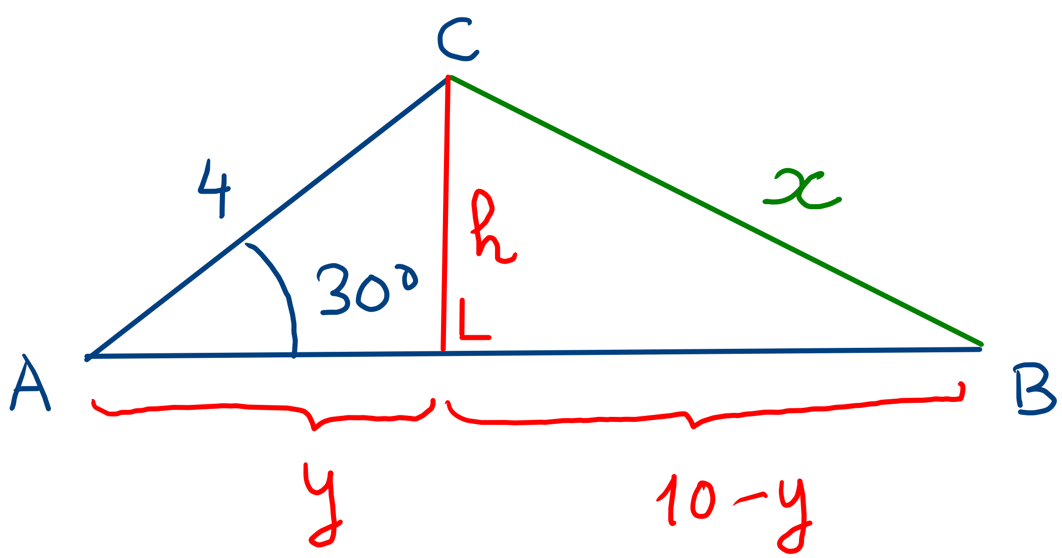
La solution de l'équation :

$$\begin{cases} x = 180° + k \cdot 360° \\ x = -112{,}62° + k \cdot 360° \end{cases}, \quad k \in \mathbb{Z}$$



sin
cos
I
II
III
IV
(+ ; +)
(− ; +)
(− ; −)
(+ ; −)

Trigonométrie dans le Δ quelconque



<u>Donnée</u> :

$AB = 10$

$AC = 4$

$\alpha = 30°$

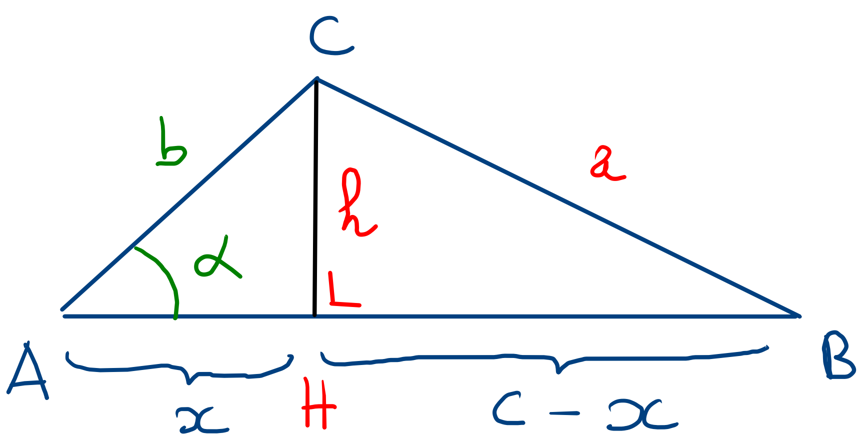
<u>But</u> : Calculer BC

$$\cos(30°) = \frac{y}{4} \Rightarrow y = 4 \cdot \cos(30°) = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

$$\begin{cases} h^2 = 4^2 - y^2 \\ h^2 = x^2 - (10 - y)^2 \end{cases}$$

On finit en calculant $h = 2$ et finalement x ...

Théorème du cosinus (partie 1)



Donnée : b, c, α

But : calculer a

1er cas : la hauteur issue de C tombe entre A et B

$$\cos(\alpha) = \frac{x}{b} \Rightarrow x = b \cdot \cos(\alpha)$$

$$\begin{cases} h^2 = b^2 - x^2 \\ h^2 = a^2 - (c-x)^2 = a^2 - c^2 + 2cx - x^2 \end{cases}$$

Donc $b^2 - \cancel{x^2} = a^2 - c^2 + 2cx - \cancel{x^2}$

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos(\alpha)}$$